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ANALYZING SCHRODINGER EQUATION FOR A PARTICLE IMPINGING ON VERTICAL POTENTIAL STEP BY MATRIX METHOD

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ABSTRACT

Quantum mechanics has been perceived as a requisite ingredient in the curricula of Physics, Chemistry, and Electrical Engineering. By applying Matrix Method, we will analyse Schrodinger's equation for a particle impinging on the vertical potential step which is generally done by different algebraic and analytical methods. The matrix method has been applied successfully in science and engineering problems. This paper demonstrates the use of the Matrix Method for analyzing Schrodinger's equation to find the reflection and transmission coefficients for a particle impinging on the vertical potential step.

Keywords: Matrix Method, Schrodinger Equation, vertical Potential Step.

INTRODUCTION

The applications of time-independent Schrodinger's equation are generally analyzed by different algebraic and analytical methods [1], [2], [3]. The matrix method has been applied successfully in science and engineering problems [4], [5], [6], [7], [8], [9], [10], [11], [12]. This paper puts forward the Matrix Method for analyzing one of the applications of Schrodinger's equation to a particle impinging on the vertical potential step and obtains the reflection and transmission coefficients. As we know for a square matrix B [5], [6], [7], [8] of order n with elements a_{ij}, we get a column matrix Z and a constant λ such that BZ = λZ or $|B - \lambda I|$Z = 0. This represents a matrix equation which results in n homogeneous linear equations [9], [10], [11], [12] having a non–trivial solution only if $|B - \lambda I|$ = 0. On expand $|B - \lambda I|$, we will get nth degree equation in λ, known as the characteristic equation of B, whose roots i.e. λ_i $(where\ i = 1, 2, 3, \ldots\ldots n)$ are called Eigenvalues, and corresponding to each Eigenvalue there is a non-zero solution Z = $\begin{bmatrix} z_1 \\ z_2 \\ \ldots \\ z_n \end{bmatrix}$ known as Eigenvector [4-12].

MATERIAL AND METHOD

The one-dimensional time-independent Schrodinger's equation [13, 14] is given by

$$\varphi''(y) + \frac{2m}{\hbar^2}\{E - V\}\varphi(y) = 0 \ldots (1)$$

Vertical Potential Step

Consider a particle of mass m and total energy E moving from a region of zero potential to a region of constant potential. The vertical Potential step is defined [13], [14] as V (y) = 0 for y < 0 and V (y) = for y > 0. If $\varphi_L(y)$ and $\varphi_R(y)$ are the wave functions to the left and the right side of the vertical potential step at y = 0, then at y = 0,

$\varphi_R(0) = \varphi_L(0) =$ g (say) and

$\varphi_R'(0) = \varphi_L'(0) =$ f (say),

where g and f are constants.

In the region, y < 0, V (y) = 0, therefore, the Schrodinger equation is written as

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$\varphi_L''(y) + k_1^2\varphi_L(y) = 0$ …… (2),

where $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$

Let us substitute

$\varphi_L(y) = \varphi_{L_1}(y)$ ….. (3)

And

$D_y\varphi_{L_1}(y) = \varphi_{L_2}(y)$… (4)

We can rewrite equation (2) as

$D_y\varphi_{L_2}(y) + k_1^2\varphi_{L_1}(y) = 0$

Or

$D_y\varphi_{L_2}(y) = -k_1^2\varphi_{L_1}(y)$ … …. (5)

Differential equations (4) and (5) can be written in single matrix form [4], [5] as

$$D_y\begin{bmatrix}\varphi_{L_1}(y)\\ \varphi_{L_2}(y)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -k_1^2 & 0\end{bmatrix}\begin{bmatrix}\varphi_{L_1}(y)\\ \varphi_{L_2}(y)\end{bmatrix}$$

The characteristic equation [6], [7] of $\begin{bmatrix}0 & 1\\ -k_1^2 & 0\end{bmatrix}$ is

$$\begin{vmatrix}0-\lambda & 1\\ -k_1^2 & 0-\lambda\end{vmatrix} = 0$$

Solving the determinant, we get

$\lambda^2 + k_1^2 = 0$

Or

$\lambda = \pm ik_1$ … (6)

Now the Eigenvector for $\lambda = ik_1$ is given by

$$\begin{bmatrix}0-ik_1 & 1\\ -k_1^2 & 0-ik_1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

Applying elementary transformation R2 → R2 + ik_1 R1, we can write

$$\begin{bmatrix}-ik_1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

This results

$-ik_1y_1 + y_2 = 0$

Or

$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1\\ ik_1\end{bmatrix}$….. (7)

And the Eigenvector for $\lambda = -ik_1$ is given by

$$\begin{bmatrix}0+ik_1 & 1\\ -k_1^2 & 0+ik_1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

Applying elementary transformation R2 → R2 $-ik_1$ R1, we can write

$$\begin{bmatrix}ik_1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

This results

$ik_1y_1 + y_2 = 0$

Or

$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1\\ -ik_1\end{bmatrix}$… (8)

The matrix of Eigenvectors is $\begin{bmatrix}1 & 1\\ ik_1 & -ik_1\end{bmatrix}$.

Let P = $\begin{bmatrix}1 & 1\\ ik_1 & -ik_1\end{bmatrix}$, then the inverse matrix of P [8], [9] is given by

$P^{-1} = \begin{bmatrix}\frac{1}{2} & \frac{1}{2ik_1}\\ \frac{1}{2} & \frac{-1}{2ik_1}\end{bmatrix}$…… (9)

To find $Pe^{\lambda y}P^{-1}$,

$$Pe^{\lambda y}P^{-1} = \begin{bmatrix}1 & 1\\ ik_1 & -ik_1\end{bmatrix}\begin{bmatrix}e^{ik_1y} & 0\\ 0 & e^{-ik_1y}\end{bmatrix}\begin{bmatrix}\frac{1}{2} & \frac{1}{2ik_1}\\ \frac{1}{2} & \frac{-1}{2ik_1}\end{bmatrix}$$

$$= \begin{bmatrix}e^{ik_1y} & e^{-ik_1y}\\ ik_1e^{ik_1y} & -ik_1e^{-ik_1y}\end{bmatrix}\begin{bmatrix}\frac{1}{2} & \frac{1}{2ik_1}\\ \frac{1}{2} & \frac{-1}{2ik_1}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{2}(e^{ik_1y} + e^{-ik_1y}) & \frac{1}{2ik_1}(e^{ik_1y} - e^{-ik_1y})\\ \frac{ik_1}{2}(e^{ik_1y} - e^{-ik_1y}) & \frac{1}{2}(e^{ik_1y} + e^{-ik_1y})\end{bmatrix}$$

$$= \begin{bmatrix}\cos k_1y & \frac{1}{k_1}\sin k_1y\\ -k_1\sin k_1y & \cos k_1y\end{bmatrix} \text{…… (10)}$$

Applying initial condition i.e. $\varphi_{L_1}(0) = g$ and $D_y\varphi_L(0) = f$, we can write

$$\begin{bmatrix}\varphi_{L_1}(y)\\ \varphi_{L_2}(y)\end{bmatrix} = \begin{bmatrix}\cos k_1y & \frac{1}{k_1}\sin k_1y\\ -k_1\sin k_1y & \cos k_1y\end{bmatrix}\begin{bmatrix}g\\ f\end{bmatrix}$$

Or

$$\begin{bmatrix}\varphi_{L_1}(y)\\ \varphi_{L_2}(y)\end{bmatrix} = \begin{bmatrix}g\cos k_1y + \frac{f}{k_1}\sin k_1y\\ -gk_1\sin k_1y + f\cos k_1y\end{bmatrix} \text{… (11)}$$

This gives

$\varphi_L(y) = \varphi_{L_1}(y) = g\cos k_1y + \frac{f}{k_1}\sin k_1y$…. (12)

Or

$$\varphi_L(y) = g\,\frac{e^{ik_1y} + e^{-ik_1y}}{2} + \frac{f}{k_1}\,\frac{e^{ik_1y} - e^{-ik_1y}}{2i}$$

Or

$\varphi_L(y) = \left(\frac{g}{2} - i\frac{f}{2k_1}\right)e^{ik_1y} + \left(\frac{g}{2} + i\frac{f}{2k_1}\right)e^{-ik_1y}$… (13)

In (13), $\left(\frac{g}{2} - i\frac{f}{2k_1}\right)e^{ik_1y}$ *and* $\left(\frac{g}{2} + i\frac{f}{2k_1}\right)e^{-ik_1y}$ represent the incident and the reflected waves in the region y < 0 i.e.

$\varphi_i(y) = \left(\frac{g}{2} - i\frac{f}{2k_1}\right)e^{ik_1y}$….. (14)

and

$\varphi_r(y) = \left(\frac{g}{2} + i\frac{f}{2k_1}\right)e^{-ik_1y}$ ….. (15)

Now, in the region, y > 0, two possibilities arise: either V (y) = $V_0 < E$ or V (y) = $V_0 > E$.

Case I: E > V_0

In this case, in the region, y > 0, V (y) = $V_0 < E$, therefore, the Schrodinger equation is written as

$[\varphi_R''(y)] + k_2^2[\varphi_R(y)] = 0$ …. (16), where $k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$ *is real*.

Let us substitute

$\varphi_R(y) = \varphi_{R_1}(y)$ …. (17)

And

$D_y\varphi_{R_1}(y) = \varphi_{R_2}(y)$… (18)

We can rewrite equation (16) as

$D_y\varphi_{R_2}(y) + k_2^2\varphi_{R_1}(y) = 0$

Or

$D_y\varphi_{R_2}(y) = -k_2^2\varphi_{R_1}(y)$

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Differential equations (18) and (19) can be written in single matrix form [10], [11] as

$$D_y \begin{bmatrix} \varphi_{R_1}(y) \\ \varphi_{R_2}(y) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_2^2 & 0 \end{bmatrix} \begin{bmatrix} \varphi_{R_1}(y) \\ \varphi_{R_2}(y) \end{bmatrix}$$

The characteristic equation of $\begin{bmatrix} 0 & 1 \\ -k_2^2 & 0 \end{bmatrix}$ is

$$\begin{vmatrix} 0-\lambda & 1 \\ -k_2^2 & 0-\lambda \end{vmatrix} = 0$$

Solving the determinant, we get

$$\lambda^2 + k_2^2 = 0$$

Or

$$\lambda = \pm i k_2 \ ..... (20)$$

Now the Eigenvector for $\lambda = ik_2$ is given by

$$\begin{bmatrix} 0-ik_2 & 1 \\ -k_2^2 & 0-ik_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying elementary transformation $R_2 \to R_2 + ik_2 R_1$, we can write

$$\begin{bmatrix} -ik_2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This results

$$-ik_2 y_1 + y_2 = 0$$

Or

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ ik_2 \end{bmatrix} ...... (21)$$

And the Eigenvector for $\lambda = -ik_2$ is given by

$$\begin{bmatrix} 0+ik_2 & 1 \\ -k_2^2 & 0+ik_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying elementary transformation $R_2 \to R_2 - ik_2 R_1$, we can write

$$\begin{bmatrix} ik_2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This results

$$ik_2 y_1 + y_2 = 0$$

Or

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -ik_2 \end{bmatrix} ... (22)$$

The matrix of Eigenvectors [12] is $\begin{bmatrix} 1 & 1 \\ ik_2 & -ik_2 \end{bmatrix}$.

Let P = $\begin{bmatrix} 1 & 1 \\ ik_2 & -ik_2 \end{bmatrix}$, then the inverse matrix of P is given by

$$P^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2ik_2} \\ \frac{1}{2} & \frac{-1}{2ik_2} \end{bmatrix} ..... (23)$$

To find $Pe^{\lambda y}P^{-1}$,

$$Pe^{\lambda y}P^{-1} = \begin{bmatrix} 1 & 1 \\ ik_2 & -ik_2 \end{bmatrix} \begin{bmatrix} e^{ik_2 y} & 0 \\ 0 & e^{-ik_2 y} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2ik_2} \\ \frac{1}{2} & \frac{-1}{2ik_2} \end{bmatrix}$$

$$= \begin{bmatrix} e^{ik_2 y} & e^{-ik_2 y} \\ ik_2 e^{ik_2 y} & -ik_2 e^{-ik_2 y} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2ik_2} \\ \frac{1}{2} & \frac{-1}{2ik_2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}(e^{ik_2 y} + e^{-ik_2 y}) & \frac{1}{2ik_2}(e^{ik_2 y} - e^{-ik_2 y}) \\ \frac{ik_2}{2}(e^{ik_2 y} - e^{-ik_2 y}) & \frac{1}{2}(e^{ik_2 y} + e^{-ik_2 y}) \end{bmatrix}$$

$$= \begin{bmatrix} \cos k_2 y & \frac{1}{k_2}\sin k_2 y \\ -k_2 \sin k_2 y & \cos k_2 y \end{bmatrix} ... (24)$$

Applying initial condition i.e. $\varphi_{R_1}(0) = g$ and $D_y \varphi_R(0)$ = f, we can write

$$\begin{bmatrix} \varphi_{R_1}(y) \\ \varphi_{R_2}(y) \end{bmatrix} = \begin{bmatrix} \cos k_2 y & \frac{1}{k_2}\sin k_2 y \\ -k_2 \sin k_2 y & \cos k_2 y \end{bmatrix} \begin{bmatrix} g \\ f \end{bmatrix}$$

Or $\begin{bmatrix} \varphi_{R_1}(y) \\ \varphi_{R_2}(y) \end{bmatrix} = \begin{bmatrix} g\cos k_2 y + \frac{f}{k_2}\sin k_2 y \\ -gk_2 \sin k_2 y + f\cos k_2 y \end{bmatrix}$

This gives

$$\varphi_R(y) = g\cos k_2 y + \frac{f}{k_2}\sin k_2 y \ .... (25)$$

Or

$$\varphi_R(y) = g\,\frac{e^{ik_2 y} + e^{-ik_2 y}}{2} + \frac{f}{k_2}\,\frac{e^{ik_2 y} - e^{-ik_2 y}}{2i}$$

Or

$$\varphi_R(y) = \left(\frac{g}{2} - i\frac{f}{2k_2}\right)e^{ik_2 y} + \left(\frac{g}{2} + i\frac{f}{2k_2}\right)e^{-ik_2 y} ... (26)$$

In (26), $\left(\frac{g}{2} - i\frac{f}{2k_2}\right)e^{ik_2 y}$ represents the transmitted wave in the region y > 0 i.e.

$$\psi_t(y) = \left(\frac{g}{2} - i\frac{f}{2k_2}\right)e^{ik_2 y} \ .... (27)$$

Since $\left(\frac{g}{2} + i\frac{f}{2k_2}\right)$ represents the coefficient of a beam incident from right on the vertical potential step, which is not physical, therefore,

$$\left(\frac{g}{2} + i\frac{f}{2k_2}\right) = 0$$

Or $f = ik_2 g$ ...... (28)

Using (28), we can write

$$\varphi_i(y) = \frac{g}{2}\left(1 + \frac{k_2}{k_1}\right)e^{ik_1 y} ... (29),$$

$$\varphi_r(y) = \frac{g}{2}\left(1 - \frac{k_2}{k_1}\right)e^{-ik_1 y} \ .... (30)$$

And $\varphi_t(y) = ge^{ik_2 y}$ ... (31)

The quantum mechanical reflection coefficient R is given by

R $= \frac{v_1 \varphi_r \varphi_r^*}{v_1 \varphi_i \varphi_i^*}$, where v_1 is the velocity in y < 0.

Using (29) and (30) and simplifying, we get

$$R = \left[\frac{k_1 - k_2}{k_1 + k_2}\right]^2 ...... (32)$$

The quantum mechanical transmission coefficient T is given by

T $= \frac{v_2 \varphi_t \varphi_t^*}{v_1 \varphi_i \varphi_i^*}$, where v_2 is the velocity in z < 0.

Using (29) and (31) and simplifying, we get

$$T = \frac{4v_2}{v_1\left(1 + \frac{k_2}{k_1}\right)^2} \ .... (33)$$

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Using the relation $\frac{p_2}{p_1} = \frac{mv_2}{mv_1} = \frac{\hbar k_2}{\hbar k_1}$,

Or $\frac{v_2}{v_1} = \frac{k_2}{k_1}$ and simplifying, we get

$T = \frac{4k_1 k_2}{(k_1+k_2)^2}$ ..... (34)

On adding equations, (32) and (34), we can find that the sum of R and T is one.

Case II: E < V_0

In this case, in the region, y < 0, the solution remains the same.

In the region, y > 0, V (y) = $V_0 < E$, therefore, the Schrodinger equation is written as

$[\varphi_R''(y)] + k_3^2[\varphi_R(y)] = 0..(35)$, where $k_3 = i\sqrt{\frac{2m(V_0-E)}{\hbar^2}} = ik$ *is complex*; $k = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$.

Let us substitute

$\varphi_R(y) = \varphi_{R_1}(y)$

And $D_y\varphi_{R_1}(y) = \varphi_{R_2}(y)$... (36)

We can rewrite equation (35) as

$D_y\varphi_{R_2}(y) + k_3^2\varphi_{R_1}(y) = 0$

Or $D_y\varphi_{R_2}(y) = -k_3^2\varphi_{R_1}(y)$ ... ... (37)

Differential equations (36) and (37) can be written in single matrix form as

$D_y\begin{bmatrix}\varphi_{R_1}(y)\\ \varphi_{R_2}(y)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -k_3^2 & 0\end{bmatrix}\begin{bmatrix}\varphi_{R_1}(y)\\ \varphi_{R_2}(y)\end{bmatrix}$

The characteristic equation of $\begin{bmatrix}0 & 1\\ -k_3^2 & 0\end{bmatrix}$ is

$\begin{vmatrix}0-\lambda & 1\\ -k_3^2 & 0-\lambda\end{vmatrix} = 0$

Solving the determinant, we get

$\lambda^2 + k_3^2 = 0$

Or

$\lambda = \pm ik_3$ ... (38)

Now the Eigen vector for $\lambda = ik_2$ is given by

$\begin{bmatrix}0-ik_3 & 1\\ -k_3^2 & 0-ik_3\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$

Applying elementary transformation R₂ → R₂ + ik_3 R₁, we can write

$\begin{bmatrix}-ik_3 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$...... (39)

This results

$-ik_3y_1 + y_2 = 0$

Or

$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1\\ ik_3\end{bmatrix}$..... (40)

And the Eigen vector for $\lambda = -ik_2$ is given by

$\begin{bmatrix}0+ik_3 & 1\\ -k_3^2 & 0+ik_3\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$

Applying elementary transformation R₂ → R₂ - ik_3 R₁, we can write

$\begin{bmatrix}ik_3 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$

This results

$ik_3y_1 + y_2 = 0$

Or

$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1\\ -ik_3\end{bmatrix}$.... (41)

The matrix of Eigen vectors is $\begin{bmatrix}1 & 1\\ ik_3 & -ik_3\end{bmatrix}$.

Let P = $\begin{bmatrix}1 & 1\\ ik_3 & -ik_3\end{bmatrix}$, then the inverse matrix of P is given by

$P^{-1} = \begin{bmatrix}\frac{1}{2} & \frac{1}{2ik_3}\\ \frac{1}{2} & \frac{-1}{2ik_3}\end{bmatrix}$...... (42)

To find $Pe^{\lambda y}P^{-1}$,

$Pe^{\lambda y}P^{-1} = \begin{bmatrix}1 & 1\\ ik_3 & -ik_3\end{bmatrix}\begin{bmatrix}e^{ik_3y} & 0\\ 0 & e^{-ik_3y}\end{bmatrix}\begin{bmatrix}\frac{1}{2} & \frac{1}{2ik_3}\\ \frac{1}{2} & \frac{-1}{2ik_3}\end{bmatrix}$

$= \begin{bmatrix}e^{ik_3y} & e^{-ik_3y}\\ ik_3e^{ik_3y} & -ik_2e^{-ik_3y}\end{bmatrix}\begin{bmatrix}\frac{1}{2} & \frac{1}{2ik_3}\\ \frac{1}{2} & \frac{-1}{2ik_3}\end{bmatrix}$

$= \begin{bmatrix}\frac{1}{2}(e^{ik_3y} + e^{-ik_3y}) & \frac{1}{2ik_2}(e^{ik_3y} - e^{-ik_3y})\\ \frac{ik_3}{2}(e^{ik_3y} - e^{-ik_3y}) & \frac{1}{2}(e^{ik_3y} + e^{-ik_3y})\end{bmatrix}$

$= \begin{bmatrix}\cos k_2y & \frac{1}{k_3}\sin k_3y\\ -k_3\sin k_3y & \cos k_3y\end{bmatrix}$...... (43)

Applying initial condition [1-4] i.e. $\varphi_{R_1}(0) = g$ and $D_y\varphi_R(0)$ = f, we can write

$\begin{bmatrix}\varphi_{R_1}(y)\\ \varphi_{R_2}(y)\end{bmatrix} = \begin{bmatrix}\cos k_3y & \frac{1}{k_3}\sin k_3y\\ -k_3\sin k_3y & \cos k_3y\end{bmatrix}\begin{bmatrix}g\\ f\end{bmatrix}$

Or

$\begin{bmatrix}\varphi_{R_1}(y)\\ \varphi_{R_2}(y)\end{bmatrix} = \begin{bmatrix}g\cos k_3y + \frac{f}{k_3}\sin k_3y\\ -gk_3\sin k_3y + f\cos k_3y\end{bmatrix}$

$\varphi_R(y) = g\cos k_3y + \frac{f}{k_2}\sin k_3y$ .... (44)

Or

$\varphi_R(y) = g\frac{e^{ik_3y} + e^{-ik_3y}}{2} + \frac{f}{k_3}\frac{e^{ik_3y} - e^{-ik_3y}}{2i}$

Or

$\varphi_R(y) = \left(\frac{g}{2} - i\frac{f}{2k_3}\right)e^{ik_3y} + \left(\frac{g}{2} + i\frac{f}{2k_3}\right)e^{-ik_3y}$ ... (45)

In (45), $\left(\frac{g}{2} - i\frac{f}{2k_3}\right)e^{ik_3y}$ represents the transmitted wave in the region y > 0 i.e.

$\varphi_t(y) = \left(\frac{g}{2} - i\frac{f}{2k_3}\right)e^{ik_3y}$ .... (46)

Since $\left(\frac{g}{2} + i\frac{f}{2k_3}\right)$ represents the coefficient of a beam incident from right on the vertical potential step, which is not physical, therefore,

$\left(\frac{g}{2} + i\frac{f}{2k_3}\right) = 0$

Or $f = ik_3g$ ...... (47)

Using (47), we can write

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$\varphi_i(y) = \frac{g}{2}\left(1 + \frac{k_3}{k_1}\right) e^{ik_1 y}$ ... (48)

$\varphi_r(y) = \frac{g}{2}\left(1 - \frac{k_3}{k_1}\right) e^{-ik_1 y}$ ..... (49)

And $\varphi_t(y) = g e^{ik_3 y}$ ...... (50)

The quantum mechanical reflection coefficient R is given by

$$R = \frac{v_1 \varphi_r \varphi_r^*}{v_1 \varphi_i \varphi_i^*}$$

Using (48) and (49) and simplifying, we get

R = 1 ...... (51)

The quantum mechanical transmission coefficient T is given by

$$T = \frac{v_2 \varphi_t \varphi_t^*}{v_1 \varphi_i \varphi_i^*}$$

Using (48) and (50) and simplifying, we get

$$T = \frac{4v_2}{v_1[1+\left(\frac{k}{k_1}\right)^2]} \quad ..... (52)$$

Using the relation $\frac{v_2}{v_1} = \frac{k_3}{k_1}$ or $\frac{v_2}{v_1} = i\frac{k}{k_1}$ and simplifying, we get

$$T = \frac{4ik}{k_1[1+\left(\frac{k}{k_1}\right)^2]}$$

Or

$Real(T) = 0$ ..... (53)

On adding (51) and (53), we can find that R + T = 1.

CONCLUSION

In this paper, Schrodinger's equation for a particle impinging on vertical potential step has been successfully analyzed by Matrix Method to find the reflection and transmission coefficients and revealed that the application of the Matrix Method is also effective and simple. The results obtained [1], [2], [3], [7], [13], [14] are the same as those obtained with other methods.

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