ISSN: 2455-7064

Contents lists available at http://www.albertscience.com

ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS)

Volume 4, Issue 1, 2020, 42-44

ANALYSIS OF VERY LONG COLUMNS WITH LOW BUCKLING LOAD VIA MEANS OF LAPLACE TRANSFORM

[1]Rohit Gupta, [2†]Rahul Gupta, [3]Dinesh Verma

[1&2] Lecturer of Physics, Department of Applied Sciences, Yogananda College of Engineering and Technology, Jammu, India.
[3]Professor, Department of Mathematics, NIILM University, Kaithal (Haryana), India.

ARTICLE INFO

Research Article History

Received: 16th October, 2020
Accepted: 30th October, 2020

Corresponding Author:
† Mr. Rahul Gupta

Email:guptara702@gmail.com

Lecturer of Physics, Department of Applied Sciences, Yogananda College of Engineering and Technology, Jammu, India.

ABSTRACT

Columns are one of the vertical compression members of a structure used in building frames and are liable to buckle and fail under relatively small axial loads. These are considerably longer in comparison with their lateral dimensions and hence buckle when the axial load approaches a certain critical value known as critical buckling load. The Laplace transform has been used in solving initial value problems in science, engineering, and technology. In this paper, it is presented for analysis of very long columns with low buckling axial loads which is generally done by applying ordinary algebraic and calculus means.

Keywords: Euler's theory, Laplace transforms, Long Columns, buckling load.

INTRODUCTION

Buckling is one of the failures of a structure carrying the load. Columns being slender, deflect laterally when compressed, and buckle when the axial load approaches a certain critical value known as critical buckling load [1]. It has been noted that the columns fail entirely due to low buckling axial loads [2], [3]. Generally, the analysis of very long columns with low buckling axial loads is done by applying ordinary algebraic and calculus means [1], [2], [3]. The Laplace transform has been used in solving initial value problems in science, engineering and technology [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]. In this paper, it is applied for the analysis of very long columns with low buckling axial loads.

BASIC DEFINITION

The Laplace transform [4], [5], [6], [7] of $h(x)$ is defined as $L[h(x)] = \int_0^\infty e^{-px} h(x)dx = H(p)$, provided that the integral is convergent for some value of real or complex parameter p.

The Laplace transform [8], [9], [10], [11], [12], [13] of some derivatives of $h(x)$ are given by

$L\{h'(x)\} = pH(p) - h(0)$,

$L\{h''(x)\} = p^2H(p) - ph(0) - h'(0)$,

$L\{h'''(x)\} = p^3H(p) - p^2h(0) - ph'(0) - h''(0)$,

and so on.

MATERIAL AND METHOD

Considering a very long vertical column AB (with lower end B) of length 'a' and having uniform cross-section. Let 'y' be the lateral deflection of the column's section at height 'x'. Now we will discuss four different cases:

Case-I: When both ends A and B of the column are pinned or hinged

In this case, the bending moment [14], [15] at the section is given by

$\ddot{y}(x) + k^2y(x) = 0$........ (1), where $k = \sqrt{\frac{P}{YI}}$.

Taking Laplace Transform of (1), we get

$L[\ddot{y}(x)] + k^2L[y(x)] = 0$

This equation gives

$p^2\bar{y}(p) - py(0) - \dot{y}(0) + k^2\bar{y}(p) = 0$....... (2)

Put $y(0) = 0$, (5) becomes,

$p^2\bar{y}(p) - \dot{y}(0) + k^2\bar{y}(p) = 0$

Or

$p^2\bar{y}(p) + k^2\bar{y}(p) = \dot{y}(0)$ ... ... (3)

Put $\dot{y}(0) = A$, (3) becomes

Doi: 10.2016-53692176; DOI Link :: http://doi-ds.org/doilink/11.2020-76488885/

$p^2\bar{y}(p) + k^2\bar{y}(p) = A$

Or

$\bar{y}(p) = \frac{A}{(p^2+k^2)}$ …….. (4)

Taking inverse Laplace transforms [4], [5], [6] of equation (4), we get

$y(x) = \frac{A}{k}\sin(kx)$ …………… (5)

Put $y(a) = 0$, (5) gives

$\sin(ka) = 0$

Or

$ka = n\pi$, where n is an integer greater than equal to zero.

Or

$k = \frac{n\pi}{a}$ ……… (6)

The least practical value of n is 1, therefore considering n = 1, we have

$k = \frac{\pi}{a}$

Or

$$\sqrt{\frac{P}{YI}} = \frac{\pi}{a}$$

Or $P = \frac{\pi^2 YI}{a^2}$ ……… (7)

This equation (7) is Euler's formula for critical buckling load for a very long column which is pinned at both ends.

Case-II: When lower end B of the column is fixed and the other end A is free

In this case, the bending moment [15], [16] at the section is given by

$\ddot{y}(x) + k^2[d - y(x)] = 0$…… (8), where'd' is the deflection at the free end A due to buckling load.

Taking Laplace Transform [7], [8], [9] of (8), we get

$L[\ddot{y}(x)] + k^2 L[y(x) - d] = 0$

This equation gives

$p^2\bar{y}(p) - py(0) - \dot{y}(0) + k^2\bar{y}(p) = \frac{k^2 d}{p}$ ... (9)

Put $y(0) = 0$ and $\dot{y}(0) = 0$ as the slope at x = 0 is zero, (9) becomes,

$p^2\bar{y}(p) + k^2\bar{y}(p) = \frac{k^2 d}{p}$

Or

$p^2\bar{y}(p) + k^2\bar{y}(p) = \frac{k^2 d}{p}$

Or

$\bar{y}(p) = \frac{k^2 d}{p\,(p^2+k^2)}$

Or

$\bar{y}(p) = \frac{d}{p} - \frac{dp}{(p^2+k^2)}$…….. (10)

Taking inverse Laplace transforms [10], [11] of (10), we get

$y(x) = d - d\cos(kx)$ ……… (11)

Applying the condition: $y(a) = d$, (11) gives

$d = d - d\cos(ka)$

Or

$\cos(ka) = 0$

Or

$ka = \frac{(2n-1)\pi}{2}$, where n is an integer greater than equal to zero.

Or

$k = \frac{(2n-1)\pi}{2a}$

The least practical value of n is 1, therefore considering n = 1, we have

$k = \frac{\pi}{2a}$

Or

$$\sqrt{\frac{P}{YI}} = \frac{\pi}{2a}$$

Or

$P = \frac{\pi^2 YI}{4a^2}$ ……… (12)

This equation (12) is the Euler's formula for critical buckling load for the very long column whose lower end is fixed and the upper end is free.

Case-III: When both the ends A and B of the column are fixed

In this case, the bending moment [1, 2] at the section is given by

$\ddot{y}(x) + k^2 y(x) = M$….. (13), where $M = \frac{M_0}{EI}$, M_0 is the restraint moment at each end.

Taking Laplace Transform of equation (13), we get

$L[\ddot{y}(x)] + k^2 L[y(x)] = \frac{M}{p}$

This equation gives

$p^2\bar{y}(p) - py(0) - \dot{y}(0) + k^2\bar{y}(p) = \frac{M}{p}$ ….(14)

Put $y(0) = 0$ and $\dot{y}(0) = 0$ as the slope at x = 0 is zero, equation (14) becomes,

$p^2\bar{y}(p) + k^2\bar{y}(p) = \frac{M}{p}$

Or

$p^2\bar{y}(p) + k^2\bar{y}(p) = \frac{M}{p}$

Or

$\bar{y}(p) = \frac{M}{p\,(p^2+k^2)}$

Or

$\bar{y}(p) = \frac{M}{k^2 p} - \frac{Mp}{k^2(p^2+k^2)}$…… (15)

Taking inverse Laplace transform [12], [13] of (15), we get

$y(x) = \frac{M}{k^2} - \frac{M}{k^2}\cos(kx)$ ……. (16)

Applying the condition: $y(a) = 0$, (16) gives $\frac{M}{k^2} - \frac{M}{k^2}$

$\cos(ka) = 0$

Therefore, $\cos(ka) = 1$

Or $ka = 2n\pi$, where n is an integer greater than equal to zero.

Or $k = \frac{2n\pi}{a}$ ….. (17)

The least practical value of n is 1, therefore considering n = 1, we have

$k = \frac{2\pi}{a}$

Or

$$\sqrt{\frac{P}{YI}} = \frac{2\pi}{a}$$

Or

$P = \frac{4\pi^2 YI}{a^2}$ ……… (18)

This equation (18) is Euler's formula for critical buckling load for the very long column whose both ends are fixed.

Doi: 10.2016-53692176; DOI Link :: http://doi-ds.org/doilink/11.2020-76488885/

Case-IV: When lower end B of the column is fixed and the upper-end A is hinged or pinned

In this case, the bending moment [1], [16] at the section is given by

$\ddot{y}(x) + k^2 y(x) = H(a-x)$ ....(19),

where $H = \frac{H_0}{EI}$, H_0 is horizontal force at the fixed end B.

Taking Laplace Transform of (19), we get

$L[\ddot{y}(x)] + k^2 L[y(x)] = HL[(a-x)]$

This equation gives

$p^2\bar{y}(p) - py(0) - \dot{y}(0) + k^2\bar{y}(p) = H\,[(\frac{a}{p} - \frac{1}{p^2})]$ ..(20)

Put $y(0) = 0$ and $\dot{y}(0) = 0$ as the slope at x = 0 is zero, (20) becomes,

$p^2\bar{y}(p) + k^2\bar{y}(p) = H\,[(\frac{a}{p} - \frac{1}{p^2})]$

Or

$\bar{y}(p) = H[\frac{a}{p(p^2+k^2)} - \frac{1}{p^2(p^2+k^2)}]$

Or

$\bar{y}(p) = H[\frac{a}{k^2 p} - \frac{ap}{k^2(p^2+k^2)} - \frac{1}{k^2 p^2} + \frac{k}{k^3(p^2+k^2)}]$ ..(21)

Taking inverse Laplace transform [11], [12], [13] of (21), we get

$y(x) = H[\frac{a}{k^2} - \frac{a}{k^2}\cos(kx) - \frac{x}{k^2} + \frac{\sin kx}{k^3}]$.. (22)

Applying the condition: $y(a) = 0$, (22) gives

$H\,[\frac{a}{k^2} - \frac{a}{k^2}\cos(ka) - \frac{a}{k^2} + \frac{\sin ka}{k^3}] = 0$

Or

$[-\frac{a}{k^2}\cos(ka) + \frac{\sin ka}{k^3}] = 0$

Or

$\frac{a}{k^2}\cos(ka) = \frac{\sin ka}{k^3}$

Or

$\tan(ka) = ka$

On expanding $\tan ka$ upto 5th power of ka and solving, we get

Or

ka = 4.5 radians

Or

$\sqrt{\frac{P}{YI}}\,a = 4.5\ radians$

Or $P = \frac{20.25YI}{a^2}$

Or $P = \frac{2\pi^2 YI}{a^2}$......... (23)

This equation (23) is Euler's formula for critical buckling load for a very long column whose lower end is fixed and the upper end is pinned.

RESULT AND DISCUSSION

This paper analyzes the very long columns with low buckling axial load by applying the Laplace transform tool. A successful attempt has been made to exemplify the Laplace transform method for analyzing the very long columns with low buckling axial loads for obtaining the Euler's formula of buckling load. In all the cases discussed, we found that the critical buckling load for very long columns which are subjected to axial loads is inversely proportional to the square of the length of the column. The results obtained are the same as obtained with ordinary and calculus algebraic methods [1-3], [14-16].

REFERENCES

[1] Theory of Structures by S. Ramamrutham and R. Narayan, 9th edition. Publisher: Dhanpat Rai, 1993.

[2] Theory of structures by R.S. Khurmi, 3rd edition. Publisher: S. Chand Limited, 1977.

[3] Theory and analysis of structures, volume 2 by A.S. Arya and O.P. Jain. Publisher: Nem Chand & Bros, 1992.

[4] Theory and Problems of Laplace Transforms, Murray R. Spiegel, First edition. Publisher: McGraw-Hill Education.

[5] Rohit Gupta, Amit Pal Singh, Dinesh Verma, Flow of Heat through A Plane Wall, And Through A Finite Fin Insulated At the Tip, International Journal of Scientific & Technology Research, Volume 8 Issue 10, October 2019, pp. 125-128.

[6] Rohit Gupta, Rahul Gupta, Sonica Rajput, Laplace Transforms Approach for the Velocity Distribution and Shear Stress Distribution of a Unidirectional Laminar Flow, International Journal for Research in Engineering Application & Management (IJREAM), Vol-04, Issue-09, Dec 2018, pp. 25-29.

[7] Rahul Gupta and Rohit Gupta, Laplace Transform method for obtaining the temperature distribution and the heat flow along a uniform conducting rod connected between two thermal reservoirs maintained at different temperatures, Pramana Research Journal, Volume 8, Issue 9, 2018, pp. 47-54.

[8] Rohit Gupta, Rahul Gupta, Dinesh Verma, Laplace Transform Approach for the Heat Dissipation from an Infinite Fin Surface, Global Journal Of Engineering Science And Researches, 6(2), February 2019, pp. 96-101.

[9] Rohit Gupta, Rahul Gupta, Dinesh Verma, Eigen Energy Values and Eigen Functions of a Particle in an Infinite Square Well Potential by Laplace Transforms, International Journal of Innovative Technology and Exploring Engineering, Volume 8 Issue 3, January 2019, pp. 6-9.

[10] Rahul Gupta, Rohit Gupta, Dinesh Verma, Total scattering cross-section of Low Energy Particles Scattered by Perfectly Rigid Sphere, Compliance Engineering Journal, Volume 10, Issue 12, 2019, pp. 477-479.

[11] Rohit Gupta , Rahul Gupta, Residue approach to mathematical analysis of the moving coil galvanometer, International Journal of Advanced Trends in Engineering and Technology, Volume 4, Issue 1, 2019, pp. 6-10.

[12] Rohit Gupta, Anamika Singh, Rahul Gupta, Quantum Theory of One Dimensional Free Electron Gas By Means Of Residue Theorem, Compliance Engineering Journal, Volume 10, Issue 12, 2019, pp. 474-476.

[13] Rohit Gupta, Loveneesh Talwar, Dinesh Verma, Exponential Excitation Response of Electric Network Circuits via Residue Theorem Approach, International Journal of Scientific Research in Multidisciplinary Studies, volume 6, issue 3, March 2020, pp. 47-50.

[14] Structures: Theory and Analysis by Martin S. Williams, D Todd, first edition. Publisher: Palgrave Macmillan.

[15] Structural Analysis: In Theory and Practice by Alan Williams. Publisher: Butterworth-Heinemann, 2009.

[16] Peter Marti, Theory of Structures – Fundamentals, Framed Structures, Plates and Shells. Publisher: Wiley, Ernst & Sohn. Berlin 2013.

How to cite the article?

Rohit Gupta, Rahul Gupta, Dinesh Verma, Analysis of very long columns with low buckling load via means of Laplace transform, ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS), 4(1), 2020: 42-44.

Doi: 10.2016-53692176; DOI Link :: http://doi-ds.org/doilink/11.2020-76488885/