



RESPONSE OF A BASIC SERIES INVERTER BY THE APPLICATION OF CONVOLUTION THEOREM

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ABSTRACT

This paper presents a new approach to demonstrate the use of the convolution in obtaining the response of basic series inverter through the application of the convolution method. The response obtained by solving the governing differential equation by the application of the convolution method will provide an expression for the electric current. The response of a basic series inverter is usually obtained by algebraic and analytic methods. In this paper, the response of a basic series inverter is provided as a demonstration of the application of the convolution method.

Index Terms: Convolution, Basic series inverter, Laplace Transform, Response.

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INTRODUCTION

An inverter is an electrical circuit which performs the function of converting DC voltage (or power) into AC voltage (or power) at the desired frequency. The classification of inverters as per the connections of semiconductor devices like thyristor (SCR) and commutating elements like a capacitor (C) and an inductor (L) is series inverters, parallel inverters and bridge inverters. In a series inverter, shown in figure 1, commutating elements L and C are connected permanently in series with the load resistor R. The thyristors T_1 and T_2 are alternately turned on. The values of commutating elements L and C in series with load resistor R are such that $R < \sqrt{\frac{L}{C}}$ so that the L - C - R network forms an under-damped circuit. A series inverter can be a voltage fed inverter in which a voltage source of constant potential is connected to the L - C - R network or it can be a current fed inverter in which a current source providing a constant current is connected to the L - C - R network. An Inverter is used in the domestic installations and in the commercial installations as a source of stand by electric supply or uninterruptible power supply, in industrial installations for induction heating and for a variable speed AC drives [1]. The working of the basic series inverter is explained in three modes of operations. In the first mode of operation, the thyristor T_1 is turned on by an external gate pulse and the capacitor is assumed to be initially

charged to a potential V_{CO} . When the thyristor T_1 turned on (as it is already forward biased due to dc voltage), it starts conducting and a current I flows in the series L - C - R network. Since the L - C - R network forms an under-damped circuit; the current is not constant and is approximately sinusoidal in nature. When it is at its peak value, the voltage across the capacitor is nearly equal to supply voltage V and after that, the current starts decreasing but the voltage across the capacitor continues to increase as it is still getting charged. When the current becomes zero the voltage across capacitor is $V + V_{CO}$ with upper plate positive and lower plate negative and the thyristor T_1 is turned off because the current flowing through it becomes less than that of the holding current. In the second mode of operation, both thyristors T_1 and T_2 are in the off state and the load current is zero and the voltage across capacitor maintains at $V + V_{CO}$ whereas the voltage across L is zero. To ensure that the thyristor T_1 is completely in off state when the stored charges in it are reduced to zero, it is necessary that the time taken by the thyristor T_1 to recover its forward blocking capability must be less than the time interval elapsed between the instants T_1 is turned OFF and T_2 is turned ON. In the third mode of operation, thyristor T_2 is turned on by an external gate pulse. Since the anode of T_2 is positive with respect to cathode due to charge on the capacitor, T_2 starts conducting and the capacitor discharges through series L - C - R network and a current which is approximately

sinusoidal in nature flows through it in a direction opposite to that in the first mode of operation. This current reaches to zero when T_2 is turned off. The above sequence of modes is repeated in the next cycle when T_1 is turned on [2]. It should be ensured that the thyristor T_1 turned on only when the thyristor T_2 is completely in the off state. The frequency of the output voltage is $\omega = \frac{1}{T+2T_{off}}$ Hz, where T the time period of oscillations and T_{off} is the time elapsed between turn off of thyristor T_1 and turn on of the thyristor T_2 or turn off of thyristor T_2 [3] and turn on of the thyristor T_1 . The response of a basic series inverter is usually obtained by algebraic and analytic methods [1-3]. In this paper, a new approach is demonstrated for obtaining the response of basic series inverter through the application of the convolution method.

Definition of Laplace Transformation

The Laplace transformation [4], [5], [6] of a function $g(y)$, where $y \geq 0$, is denoted by $G(q)$ or $L\{g(y)\}$ and is defined as $L\{g(y)\} = G(q) = \int_0^\infty e^{-qy} g(y) dy$, provided that the integral exists, where q is the parameter which may be a real or complex number and L is the Laplace transform operator.

Laplace Transformation of Derivatives of a function

If the function $g(y)$, where $y \geq 0$, is having an exponential order, that is if $g(y)$ is a continuous function and is a piecewise continuous function on any interval, then the Laplace transform of derivatives [7], [8], [9], [10], [11] of $g(y)$ are as follows

$$L\{g'(y)\} = qL\{g(y)\} - g(0),$$

$$L\{g''(y)\} = q^2G(q) - qg(0) - g'(0), \text{ and so on.}$$

Convolution and Convolution Theorem

The convolution of two functions $\varphi(y)$ and $\phi(y)$ which are defined and piecewise continuous in $[0, \infty)$, is denoted by $(\varphi * \phi)(y)$ and is defined as [4, 5, 8] $(\varphi * \phi)(y) = \int_0^y \varphi(r) \phi(y-r) dr$, where $y \geq 0$.

If these functions $\varphi(y)$ and $\phi(y)$ are of exponential order, then the Laplace transform of $[(\varphi * \phi)(y)]$ is given by

$$L[(\varphi * \phi)(y)] = L[\varphi(y)] L[\phi(y)] = \bar{\varphi}(q) \bar{\phi}(q),$$

where $\bar{\varphi}(q)$ and $\bar{\phi}(q)$ are Laplace transforms of $\varphi(y)$ and $\phi(y)$ and L is Laplace transform operator. Applying inverse Laplace Transform, we can write $L^{-1}[\bar{\varphi}(q) \bar{\phi}(q)] = [(\varphi * \phi)(y)]$

METHODOLOGY

To derive the governing differential equation:

Considering a basic series inverter which consists of a series $L - C - R$ network connected to a steady excitation voltage source of constant potential V through a thyristor T_1 and a thyristor T_2 is also connected in parallel to the series $L - C - R$ network [1-3] as shown in figure 1. **In the first mode of operation of the basic series inverter**, since the thyristor T_2 is off and the thyristor T_1 is on, therefore, the equivalent circuit, in this case, is shown in figure 2.

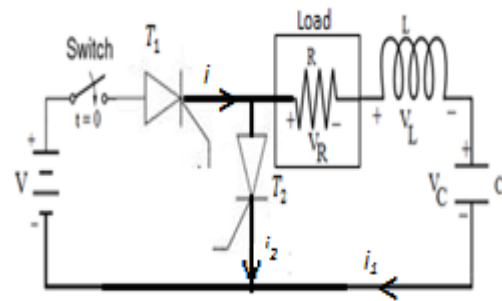


Figure 1: Basic Series Inverter

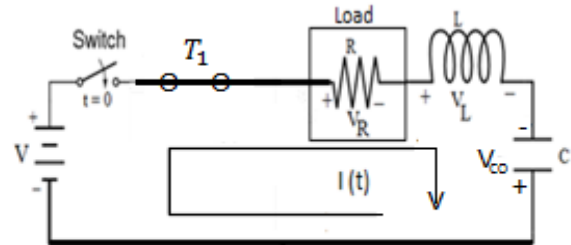


Figure 2: Equivalent Circuit (T_2 is OFF and T_1 is ON)

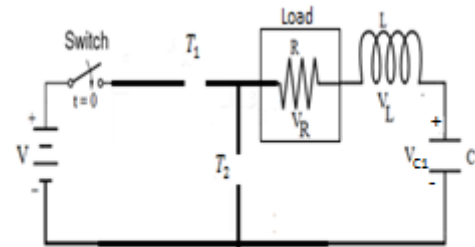


Figure 3: Equivalent Circuit (Both T_1 and T_2 are OFF)

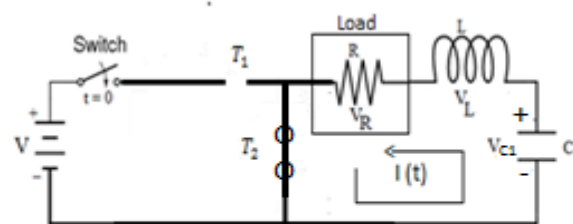


Figure 4: Equivalent Circuit (T_1 is OFF and T_2 is ON)

When the switch is closed at $t = 0$, the potential drops across the network elements are given by $V_R(t) = RI(t)$, $V_L(t) = L \frac{dI(t)}{dt}$ and $V_C(t) = \frac{q(t)}{C}$, where $q(t)$ is the charge on the capacitor at any instant of time t . In this mode of operation, the capacitor is assumed to be initially charged to a potential V_{CO} , with the upper plate having negative polarity and the lower plate having positive polarity. Therefore, the application of Kirchhoff's loop law to the loop shown in figure 2 provides

$$V_R(t) + V_L(t) + V_C(t) = V + V_{CO}$$

Or

$$RI(t) + L \frac{dI(t)}{dt} + \frac{q(t)}{C} = V + V_{CO} \dots (1) \quad \frac{d}{dt} \equiv \frac{d}{dt}$$

Differentiate equation (1), one obtained a linear homogeneous differential equation of order 2 as given below:

$$R\mathfrak{D}_t[I(t)] + L\mathfrak{D}_t^2[I(t)] + \frac{1}{C}I(t) = 0, \text{ where } I(t) = \mathfrak{D}_t[q(t)]$$

is the instantaneous electric current flowing in the series L - C - R network circuit.

$$\text{Or } L\mathfrak{D}_t^2[I(t)] + R\mathfrak{D}_t[I(t)] + \frac{1}{C}I(t) = 0$$

$$\text{Or } \mathfrak{D}_t^2[I(t)] + \frac{R}{L}\mathfrak{D}_t[I(t)] + \frac{1}{LC}I(t) = 0 \dots (2)$$

To solve equation (2), one first write the relevant boundary conditions as follows [1-3]:

Since the current through the inductor and the electric potential across the capacitor cannot be changed instantaneously, therefore, as the switch is closed at the instant $t = 0$, then $I(0) = 0$.

Since at the instant $t = 0$, $I(0) = 0$, therefore, equation (1) provides $L\mathfrak{D}_t[I(0)] = V + V_{CO}$ or $\mathfrak{D}_t[I(0)] = \frac{V + V_{CO}}{L}$.

The Laplace transform [6], [7], [9], [10], [11] of equation (2) provides $q^2\bar{I}(q) - qI(0) - D_t[I(0)] + \frac{R}{L}\{q\bar{I}(q) - I(0)\} + \frac{1}{LC}\bar{I}(q) = 0 \dots (3)$

Applying boundary conditions: $I(0) = 0$ and $\mathfrak{D}_t[I(0)] = \frac{V + V_{CO}}{L}$, equation (3) becomes,

$$q^2\bar{I}(q) - \frac{V + V_{CO}}{L} + \frac{R}{L}q\bar{I}(q) + \frac{1}{LC}\bar{I}(q) = 0$$

$$\text{Or } \bar{I}(q) \left[q^2 + \frac{R}{L}q + \frac{1}{LC} \right] = \frac{V + V_{CO}}{L}$$

$$\text{Or } \bar{I}(q) = \frac{V + V_{CO}}{L} \left[\frac{1}{q^2 + \frac{R}{L}q + \frac{1}{LC}} \right]$$

$$\text{Or } \bar{I}(q) = \frac{V + V_{CO}}{L} \left[\frac{1}{q^2 + 2q\frac{R}{2L} + \left(\frac{R}{2L}\right)^2 - \left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$\text{Or } \bar{I}(q) = \frac{V + V_{CO}}{L} \left[\frac{1}{\left(q + \frac{R}{2L}\right)^2 - \left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$\text{Or } \bar{I}(q) = \frac{V + V_{CO}}{L} \left[\frac{1}{\left(q + \frac{R}{2L}\right)^2 + \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]} \right] \dots (4)$$

According to the condition for the circuit to be under-damped, $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2$ or $\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 > 0$, therefore on

putting $\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 = \omega^2$ or $\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$ in equation (4), we can rewrite equation (4) as

$$\bar{I}(q) = \frac{V + V_{CO}}{L} \left[\frac{1}{\left(q + \frac{R}{2L}\right)^2 + \omega^2} \right]$$

$$\text{Or } \bar{I}(q) = \frac{V + V_{CO}}{L} \left[\frac{1}{\left(q + \frac{R}{2L}\right)^2 - (i\omega)^2} \right]$$

$$\text{Or } \bar{I}(q) = \frac{V + V_{CO}}{L} \left[\frac{1}{\left(q + \frac{R}{2L} - i\omega\right)\left(q + \frac{R}{2L} + i\omega\right)} \right] \dots (5)$$

Let $F(q) = \frac{1}{\left(q + \frac{R}{2L} - i\omega\right)}$ and $G(q) = \frac{1}{\left(q + \frac{R}{2L} + i\omega\right)}$,

Then the inverse Laplace transforms of these functions are given by $f(t) = e^{-\left(\frac{R}{2L} - i\omega\right)t}$ and $g(t) = e^{-\left(\frac{R}{2L} + i\omega\right)t}$.

Equation (5) can be rewritten as

$$\bar{I}(q) = \frac{V + V_{CO}}{L} [F(q) \times G(q)] \dots (6)$$

Taking inverse Laplace transform of equation (6), one can write

$$I(t) = \frac{V + V_{CO}}{L} L^{-1}[F(q) \times G(q)] \dots (7)$$

Now applying convolution theorem, we can write

$$L^{-1}[F(q) \times G(q)] = (f * g)(t) = \int_0^t f(\tau)g(t - \tau) d\tau$$

$$\text{Or } L^{-1}[F(q) \times G(q)] = \int_0^t e^{-\left(\frac{R}{2L} - i\omega\right)\tau} e^{-\left(\frac{R}{2L} + i\omega\right)(t - \tau)} d\tau$$

$$\text{Or } L^{-1}[F(q) \times G(q)] = \int_0^t e^{-\left(\frac{R}{2L} - i\omega\right)\tau - \left(\frac{R}{2L} + i\omega\right)(t - \tau)} d\tau$$

$$\text{Or } L^{-1}[F(q) \times G(q)] = e^{-\left(\frac{R}{2L} + i\omega\right)t} \int_0^t e^{2i\omega\tau} d\tau$$

$$\text{Or } L^{-1}[F(q) \times G(q)] = \frac{e^{-\left(\frac{R}{2L} + i\omega\right)t}}{2i\omega} [e^{2i\omega t}]_0^t$$

$$\text{Or } L^{-1}[F(q) \times G(q)] = \frac{e^{-\left(\frac{R}{2L} + i\omega\right)t}}{2i\omega} [e^{2i\omega t} - 1]$$

$$\text{Or } L^{-1}[F(q) \times G(q)] = \frac{e^{-\frac{R}{2L}t}}{2i\omega} [e^{i\omega t} - e^{-i\omega t}]$$

$$\text{Or } L^{-1}[F(q) \times G(q)] = \frac{e^{-\frac{R}{2L}t}}{\omega} \sin \omega t \dots (8)$$

Using equation (8) in equation (9), one obtained

$$I(t) = \frac{V + V_{CO}}{L} \frac{e^{-\frac{R}{2L}t}}{\omega} \sin \omega t$$

$$\text{Or } I(t) = \frac{V + V_{CO}}{\omega L} e^{-\frac{R}{2L}t} \sin \omega t \dots (9)$$

This equation (9) confirms that the current $I(t)$ is sinusoidal in nature with exponentially decreasing amplitude. At $\omega t = \pi$ or $t = \frac{\pi}{\omega}$, $I\left(\frac{\pi}{\omega}\right) = 0$ i.e. at the instant $t = \frac{\pi}{\omega}$, the current in the circuit becomes zero.

To find the voltage drop across the inductor, differentiating equation (9) w.r.t. t, it can be written

$$\mathfrak{D}_t[I(t)] = \frac{V + V_{CO}}{\omega L} e^{-\frac{R}{2L}t} \left[\omega \cos \omega t - \frac{R}{2L} \sin \omega t \right] \dots (10)$$

For simplifying equation (10), let us put $\frac{R}{2L} = b$, where b is known as damping constant, and $\frac{1}{LC} = \omega_o^2$, where ω_o is known as resonant frequency such that $\omega = \sqrt{\omega_o^2 - b^2}$ or $\omega_o = \sqrt{\omega^2 + b^2}$, then we can rewrite equation (10) as

$$\mathfrak{D}_t[I(t)] = \frac{V + V_{CO}}{\omega L} e^{-bt} [\omega \cos \omega t - b \sin \omega t]$$

$$\text{Or } \{\mathfrak{D}_t[I(t)]\} = \frac{V + V_{CO}}{\omega L} e^{-bt} \omega_o \left[\frac{\omega}{\omega_o} \cos \omega t - \frac{b}{\omega_o} \sin \omega t \right] \dots (11)$$

Put $\frac{\omega}{\omega_o} = \cos \varphi$ and $\frac{b}{\omega_o} = \sin \varphi$ such that $\varphi = \tan^{-1} \frac{b}{\omega}$, equation (11) becomes

$$\mathfrak{D}_t[I(t)] = \frac{V + V_{CO}}{\omega L} e^{-bt} \omega_o [\cos \varphi \cos \omega t - \sin \varphi \sin \omega t]$$

$$\text{Or } \mathfrak{D}_t[I(t)] = \frac{V + V_{CO}}{\omega L} e^{-bt} \omega_o \cos(\omega t + \varphi) \dots (12)$$

The voltage drop across the inductor is given by

$$V_L(t) = L\mathfrak{D}_t[I(t)] = \frac{V + V_{CO}}{\omega} e^{-bt} \omega_o \cos(\omega t + \varphi) \dots (13)$$

We can determine the voltage across capacitor as

$$V_R(t) + V_L(t) + V_C(t) = V$$

$$\text{Or } V_C(t) = V - V_R(t) - V_L(t)$$

$$\text{Or } V_C(t) = V - R I(t) - V_L(t) \dots (14)$$

Using equations (9) and (13) in equation (14) and simplifying, one obtained

$$V_C(t) = V - \frac{V + V_{CO}}{\omega} e^{-bt} \omega_o \cos(\omega t - \varphi) \dots (15)$$

At

$$\omega t = \pi \text{ or } t = \frac{\pi}{\omega}, V_C \left(\frac{\pi}{\omega} \right) = V - \frac{V + V_{CO}}{\omega} e^{-b \frac{\pi}{\omega}} \omega_o \cos(\pi - \varphi)$$

$$\text{Or } V_C \left(\frac{\pi}{\omega} \right) = V + \frac{V + V_{CO}}{\omega} e^{-b \frac{\pi}{\omega}} \omega_o \cos(\varphi) \dots\dots\dots (16)$$

As $\frac{\omega}{\omega_o} = \cos \varphi$, equation (16) becomes

$$\text{Or } V_C \left(\frac{\pi}{\omega} \right) = V + (V + V_{CO}) e^{-b \frac{\pi}{\omega}}$$

For convenience, let us write $V_C \left(\frac{\pi}{\omega} \right) = V_{C1}$, then

$$V_{C1} = V_C \left(\frac{\pi}{\omega} \right) = V + (V + V_{CO}) e^{-b \frac{\pi}{\omega}}$$

Or

$$V_{C1} = V_C \left(\frac{\pi}{\omega} \right) = V + (V + V_{CO}) e^{-\frac{R\pi}{2L\omega}} \dots\dots\dots (17)$$

This equation (17) provides the voltage across the capacitor at the instant $t = \frac{\pi}{\omega}$.

In the second mode of operation of the basic series inverter, as both the thyristors T_1 and T_2 are in the off state as shown in figure 3, therefore, in this mode of operation, $I(t) = 0$, $V_C(t) = V_{C1}$ and $V_L(t) = 0$.

In the third mode of operation of the basic series inverter, since the thyristor T_1 is off and the thyristor T_2 is on, therefore, the equivalent circuit, in this case, is shown in figure 4. In this case, the capacitor is initially charged to a potential V_{C1} with positive polarity on the upper plate and negative polarity on the lower plate. The direction of flow of current on this case is in opposite direction to that of current that flows in the first mode of operation of the basic series inverter [12].

The application of Kirchoff's loop law to the loop shown in figure 4 provides

$$V_R(t) + V_L(t) + V_C(t) = V_{C1}$$

Or

$$R I(t) + L \frac{dI(t)}{dt} + \frac{q(t)}{C} = V_{C1} \dots\dots\dots (18)$$

Equation (18) is similar to the equation (1). Here, on the right-hand side of equation (18), the term V_{C1} appears instead of term $V + V_{CO}$ which appeared on the right-hand side of equation (1). Hence the solution of equation (18) can be obtained in a similar manner by the convolution method as that of equation (1) and is given by

$$I(t) = \frac{V_{C1}}{\omega L} e^{-bt} \sin \omega t$$

Or

$$I(t) = \frac{V_{C1}}{\omega L} e^{-\frac{R}{2L}t} \sin \omega t \dots\dots\dots (19)$$

This equation (19) confirms that the current $I(t)$ is sinusoidal in nature with exponentially decreasing amplitude. It is clear from the equations (9) and (19) that the amplitude of current in the first mode of operation will be equal to the amplitude of current in the third mode of operation only if $V_{C1} = V + V_{CO}$.

CONCLUSION:

In this paper, an attempt is made to exemplify the convolution approach for determining the response (electric current) of basic series inverter through the application of the convolution method. This approach brings up the convolution approach as a powerful technique for determining the response of power electronic.

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