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RELATION BETWEEN EULERIAN INTEGRAL OF THE FIRST KIND AND EULER'S INTEGRAL OF SECOND KIND BY ELZAKI TRANSFORM

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ABSTRACT

In this paper we will find the relation between Eulerian integral of the first kind and Euler's integral of second kind. Euler integral of the first kind is used to determine average time of completing selected tasks in time management, and Euler's integral of second kind is one of the most widely used special functions encountered in advanced mathematics. The purpose of paper is to prove the relation between Euler integral of the first kind and Euler's integral of second kind.

Keywords: Elzaki Transform, Laplace transforms, Beta function, gamma function.

I. INTRODUCTION

Elzaki Transform approach has been applied in solving boundary value problems in most of the science and engineering disciplines [1, 2, 3, 4, 5, 6, 7, 8, 9]. It also comes out to be very effective tool find the relation between Euler integral of the first kind and the gamma function [10, 11, 12, 13, 14, 15, 16, 17, 18]. The relation between Euler integral of the first kind and the gamma function is generally obtained by definition of beta and gamma functions or by Laplace transforms method [18, 19, 20, 21, 22, 23, 24, 25, 26]. In this paper, we present a new approach called Elzaki transform approach to find the relation between Euler integral of the first kind and the gamma function.

II. DEFINITIONS

Elzaki Transform

If the function $\hbar(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of $\hbar(y)$ is given by

$$E\{\hbar(z)\} = \bar{\hbar}(p) = p\int_0^\infty e^{-\frac{z}{p}}\hbar(y)dy.$$

The Elzaki Transform [1, 2, 3,] of some of the functions are given by

- $E\{z^n\} = n!\,p^{n+2}$, $where\ n = 0,1,2,..$
- $E\{e^{az}\} = \frac{p^2}{1-ap}$,
- $E\{sinaz\} = \frac{ap^3}{1+a^2p^2}$,
- $E\{cosaz\} = \frac{ap^2}{1+a^2p^2}$,
- $E\{sinhaz\} = \frac{ap^3}{1-a^2p^2}$,
- $E\{coshaz\} = \frac{ap^2}{1-a^2p^2}$.

Definition of Beta function

The integral $\int_0^1 w^{x-1}(1-w)^{y-1}dw$, which converges for $x > 0, y > 0$ is called the beta function [1, 7] and is denoted by $\beta(x,y)$. Thus,

$$\beta(x,y) = \int_0^1 w^{x-1}(1-w)^{y-1}dw,$$

Where, $x > 0, y > 0$

Beta function is also known as Eulerian integral of first kind.

Definition of Gamma function

The gamma function [4, 5, 6,] is defined as the definite integral

$$\Gamma x = \int_0^\infty e^{-w}w^{x-1}dw,\ p > 0.$$

Gamma function is also known as Euler's integral of second kind.

Convolution Theorem [1, 2, 3,]of Elzaki

$$H(t) = \frac{1}{p}\int_0^t H_1(w).H_2(t-w)dw = H_1 * H_2$$

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$E\{H(t)\} = E\{H_1 * H_2\} = \frac{1}{p} h_1(p).h_2(p)$

$where, h_1(p) = L\{H_1(w)\}$

$and \quad h_2(p) = L\{H_2(w)\}$

III. METHODOLOGY

1. **Relation between beta and gamma function by using Elzaki transformation**

We know that by the definition of beta function

$\beta(x,y) = \int_0^1 w^{x-1}(1-w)^{y-1}dw = \frac{\Gamma x \Gamma y}{\Gamma x+y}$

$let, H(t) = \frac{1}{p}\int_0^t w^{x-1}(t-w)^{y-1}dw \ldots(1)$

$H_1(w) = w^{x-1} \; and \; H_2(w) = w^{y-1} \ldots..(2)$

By convolution theorem of Elzaki transformation,

$E\{H(t)\} = E\{H_1 * H_2\} = \frac{1}{p} h_1(p).h_2(p)$

$where, h_1(p) = L\{H_1(w)\}$

$and \quad h_2(p) = L\{H_2(w)\}$

$E\{t^{x-1}\}.E\{t^{y-1}\} = p^{x+1}\; x-1!.p^{y+1}\; y-1!$

$H(t) = E^{-1}\left[\left\{\frac{1}{p}p^{x+1}p^{y+1}\;\Gamma x\;\Gamma y\right\}\right]$

$= E^{-1}[\{\frac{1}{p}p^{x+y+2}\;\Gamma x\;\Gamma y\}]$

$= E^{-1}[\{p^{x+y+1}\;\Gamma x\;\Gamma y\}]$

From (1),

$\int_0^t w^{x-1}(t-w)^{y-1}dw \quad = \{\Gamma x\;\Gamma y\}E^{-1}\{p^{x+y+1}\}$

$\int_0^t w^{x-1}(t-w)^{y-1}dw \quad = \{\Gamma x\;\Gamma y\}\;\frac{t^{x+y-1}}{\Gamma(x+y)}$

Putting t=1

$\int_0^1 w^{x-1}(1-w)^{y-1}dw = \frac{\Gamma x \Gamma y}{\Gamma x+y}$

2. **Relation between beta and gamma function by using Laplace transformation**

$\beta(x,y) = \int_0^1 w^{x-1}(1-w)^{y-1}dw = \frac{\Gamma x \Gamma y}{\Gamma x+y}$

$let, H(t) = \int_0^t w^{x-1}(t-w)^{y-1}dw \ldots(1)$

$H_1(w) = w^{x-1} \; and \; H_2(w) = w^{y-1} \ldots..(2)$

$H(t) = \int_0^t H_1(w).H_2(t-w)dw = H_1 * H_2$

By convolution theorem of Laplace transformation,

$L\{H(t)\} = L\{H_1 * H_2\} = h_1(p).h_2(p)$

$where, h_1(p) = L\{H_1(w)\}$

$and \quad h_2(p) = L\{H_2(w)\}$

$L\{t^{x-1}\}.L\{t^{y-1}\} = \frac{\Gamma x}{p^x}.\frac{\Gamma y}{p^y} = \frac{\Gamma x\;\Gamma y}{p^{x+y}}$

$H(t) = L^{-1}\left\{\frac{\Gamma x\;\Gamma y}{p^{x+y}}\right\}$

$= \Gamma x\;\Gamma y\; L^{-1}\{\frac{1}{p^{x+y}}\}$

$= \Gamma x\;\Gamma y\; L^{-1}\{\frac{t^{x+y-1}}{\Gamma x+y}\}$

From (1),

$\int_0^t w^{x-1}(t-w)^{y-1}dw = \Gamma x\;\Gamma y\; L^{-1}\{\frac{t^{x+y-1}}{\Gamma x+y}\}$

Putting t=1

$\int_0^1 w^{x-1}(1-w)^{y-1}dw = \frac{\Gamma x\;\Gamma y}{\Gamma x+y}$

3. **Relation between beta and gamma function by the definition of gamma function:**

We know that by the definition of gamma function

$\Gamma x = \int_0^\infty e^{-w}w^{x-1}dw, \qquad p > 0.$

putting $w = u^2, dw = 2u\,du$

$\Gamma x = \int_0^\infty e^{-u^2}u^{2x-2}\;2u\,du$

$\Gamma x = 2\int_0^\infty e^{-u^2}u^{2x-1}\,du$ ....(1)

Similarly,

$\Gamma y = 2\int_0^\infty e^{-v^2}u^{2y-1}\,dv$ ....(2)

Therefore,

$\Gamma x\;\Gamma y = 4\int_0^\infty e^{-u^2}u^{2x-1}\,du\int_0^\infty e^{-v^2}u^{2y-1}\,dv$

$\Gamma x\;\Gamma y = 4\int_0^\infty\int_0^\infty e^{-(u^2+v^2)}u^{2x-1}u^{2y-1}\,dudv$

$Substituting\; u = rcos\theta \;,\; v = rsin\theta$

$Then\; dudv = rdrd\theta,$

$limit\; of\; r\; is\; 0 to\; \infty\; and\; limitof\; \theta\; is\; 0\; to\; \frac{\pi}{2}$

Hence,

$\Gamma x\;\Gamma y = 4\int_0^{\frac{\pi}{2}}\int_0^\infty e^{-r^2}r^{2(x+y)-1}cos^{2x-1}\theta sin^{2y-1}\theta\; drd\theta$

$= 4\int_0^{\frac{\pi}{2}}\int_0^\infty e^{-r^2}r^{2(x+y)-1}dr\;cos^{2x-1}\theta sin^{2y-1}\theta\; d\theta$

$\Gamma x + y[2\int_0^{\frac{\pi}{2}} cos^{2x-1}\theta sin^{2y-1}\theta\; d\theta]$

$\Gamma x\;\Gamma y = \Gamma x+y\;\beta(x,y)$

Hence,

$\beta(x,y) = \frac{\Gamma x\;\Gamma y}{\Gamma x+y}$

IV. CONCLUSION

This paper presented a new approach called Elzaki transform approach to find the relation between Eulerian integral of the first kind and Euler's integral of second kind. It has come out to be very effective tool to find the relation between Eulerian integral of the first kind and Euler's integral of second kind. We have also shown the two other known methods.

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