



ANALYSIS OF SERIES RL AND RC NETWORKS WITH SINUSOIDAL POTENTIAL SOURCES BY GUPTA TRANSFORM

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ABSTRACT

The analysis of electric networks is an essential course for electrical and electronics engineering. The response of such networks is generally obtained by the different mathematical approaches and integral transforms. This paper presents a new integral transform called Gupta Transform for obtaining the complete response of the series RL and RC networks with sinusoidal potential sources. The response obtained will provide electric current (or electric charge) flowing through the series RL and RC network circuits. In this paper, the response of the series RL and RC networks is provided as a demonstration of the application of the new integral transform called Gupta Transform.

Index Terms: Gupta Transform; Current; Series RL and RC Networks; Response.

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I. INTRODUCTION

The electric circuit of the series RL network consists of two passive electric elements: an inductor L and a resistor R , connected in series with a source of sinusoidal potential and the electric circuit of the series RC network consists of two passive electric elements: a capacitor C and a resistor R , connected in series with a source of sinusoidal potential. Such networks are used as a tuning or resonant circuit in the radio and television sets to resonate a particular frequency band from the wide range of radio frequency components, or in the chokes of luminescent tubes [1, 2, 3 & 4]. The Gupta Transform was proposed by the authors Rahul Gupta and Rohit Gupta in recent years and generally, it has been applied in different areas of science and engineering [5, 6]. The response of electrical networks is generally obtained by the different mathematical approaches like the calculus approach [1-3], convolution theorem approach [4, 7, 8], and by various integral transforms like Laplace Transform [1-3], Mohand Transform [9, 10], Aboodh Transform [11], Elzaki Transform [12, 13, 14], Rohit Transform [15], residue theorem approach [16], Matrix method [17, 18, 19, 20], etc. This paper presents the use of a new integral transform called Gupta Transform for obtaining the complete response of the series RL and RC networks with a source of sinusoidal potential.

Basics of Gupta Transform

The Gupta Transform of $g(y)$, $y \geq 0$ is defined as [5, 6 & 21]

$$\dot{R}\{g(y)\} = \frac{1}{q^3} \int_0^{\infty} e^{-qy} g(y) dy = G(q), \text{ provided that the}$$

integral is convergent. Here q may be a real or complex parameter and \dot{R} is the Gupta Transform operator.

The Gupta Transform of elementary functions are given in [5, 6 & 21].

The inverse Gupta Transform of the function $G(r)$ is denoted by $\dot{R}^{-1}\{G(r)\}$ or $g(y)$.

If we write $\dot{R}\{g(y)\} = G(r)$, then $\dot{R}^{-1}\{G(r)\} = g(y)$, where \dot{R}^{-1} is called the inverse Gupta Transform operator.

The Inverse Gupta Transform of elementary functions are given in [5, 6].

The Gupta Transform of some derivatives [5, 6] of $g(y)$ are

$$\dot{R}\{g'(y)\} = q\dot{R}\{g(y)\} - \frac{1}{q^3}g(0),$$

$$\dot{R}\{g''(y)\} = q^2\dot{R}\{g(y)\} - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0).$$

And so on.

II. FORMULATION

A. SERIES RL NETWORK CIRCUIT WITH SINUSOIDAL POTENTIAL SOURCE

We will take a series RL network to which a sinusoidal voltage source of potential $V_0 \sin \omega t$ is applied through a

key K as shown in figure 1.

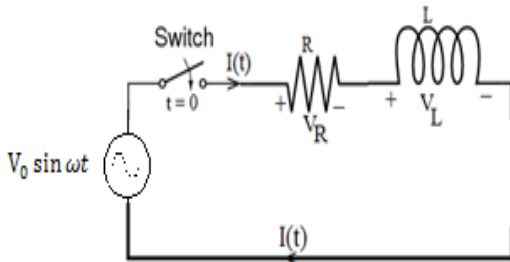


Figure 1: Series LR network with sinusoidal voltage source.

As the switch is closed at $t = 0$, the potential drops across the network elements are given by [1-4]

$$V_R(t) = I(t)R, V_L(t) = L \frac{dI(t)}{dt}$$

Therefore, the application of Kirchoff's loop law to the loop shown in figure 2 provides

$$V_R(t) + V_L(t) = V$$

Or

$$RI(t) + L \frac{dI(t)}{dt} = V_0 \sin \omega t \dots (1) \quad \frac{d}{dt} \equiv \frac{d}{dt}$$

Differentiating (1), we get a differential equation of order 2 as given below:

$$\frac{d^2 I(t)}{dt^2} + \frac{R}{L} \frac{dI(t)}{dt} = \frac{V_0 \omega}{L} \cos \omega t \dots (2)$$

Solution governing equation by Gupta Transform

To solve equation (2), we first write the initial conditions as follows [1-4, 22]:

- Since the current in the inductor and the electric potential difference across the capacitor cannot change instantaneously, therefore, as the switch is closed at the instant $t = 0$, then $I(0) = 0$.
- Since $I(0) = 0$, therefore, (1) provides $\frac{dI(0)}{dt} = 0$.
The Gupta Transform [6, 21, 22,] of (2) provides

$$q^2 \bar{I}(q) - \frac{1}{q^2} I(0) - \frac{1}{q^3} \frac{dI(0)}{dt} + \frac{R}{L} \{q \bar{I}(q) - \frac{1}{q^2} I(0)\} = \frac{V_0 \omega}{L} \frac{1}{q^2 q^2 + \omega^2} \dots (3)$$

Applying $I(0) = 0$ and $\frac{dI(0)}{dt} = 0$, (3) becomes,

$$q^2 \bar{I}(q) + \frac{R}{L} q \bar{I}(q) = \frac{V_0 \omega}{L} \frac{1}{q^2 + \omega^2}$$

Or

$$\bar{I}(q) = \frac{V_0 \omega}{L} \frac{1}{q^3 q^2 + \omega^2} \left[\frac{1}{q + \frac{\omega}{q}} \right]$$

This equation can be re written as

$$\bar{I}(q) = \frac{V_0}{L} \left\{ \left[\frac{1}{(\omega L)^2 + R^2} \left(R \frac{1}{q^3 q^2 + \omega^2} - \omega L \frac{1}{q^2 q^2 + \omega^2} \right) \right] + \left[\frac{\omega L^2}{(\omega L)^2 + R^2} \frac{1}{q^3 q + \frac{R}{L}} \right] \right\}$$

Taking inverse Gupta Transform [5], we have

$$I(t) =$$

$$\frac{V_0}{L} \left\{ \left[\frac{1}{(\omega L)^2 + R^2} (R \sin \omega t - \omega L \cos \omega t) \right] + \left[\frac{\omega L^2}{(\omega L)^2 + R^2} e^{-\left(\frac{R}{L}\right)t} \right] \right\}$$

Or

$$I(t) = \frac{V_0}{\sqrt{(\omega L)^2 + R^2}}$$

$$\left\{ \left[\frac{R}{\sqrt{(\omega L)^2 + R^2}} \sin \omega t - \frac{\omega L}{\sqrt{(\omega L)^2 + R^2}} \cos \omega t \right] + \left[\frac{\omega L}{\sqrt{(\omega L)^2 + R^2}} e^{-\left(\frac{R}{L}\right)t} \right] \right\}$$

..... (4)

Put $\frac{\omega L}{\sqrt{(\omega L)^2 + R^2}} = \sin \phi$ and $\frac{R}{\sqrt{(\omega L)^2 + R^2}} = \cos \phi$ such

that $\tan \phi = \frac{\omega L}{R}$, then we can rewrite equation (4) as

$$I(t) = \frac{V_0}{\sqrt{(\omega L)^2 + R^2}}$$

$$\left\{ [\cos \phi \sin \omega t - \sin \phi \cos \omega t] + [\sin \phi e^{-\left(\frac{R}{L}\right)t}] \right\}$$

Or

$$I(t) = \frac{V_0}{\sqrt{(\omega L)^2 + R^2}} \{ \sin(\omega t - \phi) + \sin \phi e^{-\left(\frac{R}{L}\right)t} \} \dots (5)$$

This equation (5) provides the complete response of the series $L - R$ network with a source of sinusoidal potential.

B. SERIES RC NETWORK CIRCUIT WITH SINUSOIDAL POTENTIAL SOURCE

The series R-C network circuit with a sinusoidal potential source (as shown in figure 2) is analyzed by the following equation [1-3]

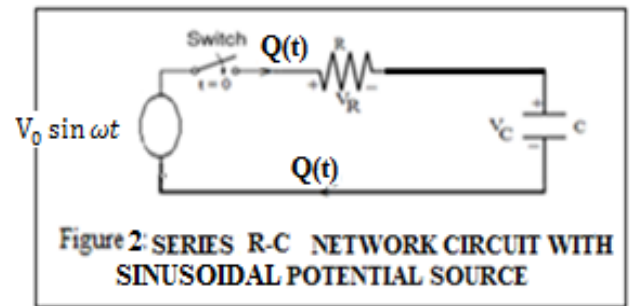


Figure 2: SERIES R-C NETWORK CIRCUIT WITH SINUSOIDAL POTENTIAL SOURCE

$$Q(t)R + \frac{Q(t)}{C} = V_0 \sin \omega t$$

Or

$$Q(t) + \frac{1}{RC} Q(t) = \frac{V_0}{R} \sin \omega t \dots (6)$$

Here, $Q(t)$ is the instantaneous charge and $Q(0) = 0$.

The Gupta Transform [6, 21] of (6) gives

$$\{q \bar{Q}(q) - \frac{1}{q^3} Q(0)\} + \frac{1}{RC} \bar{Q}(q) = \frac{V_0}{R} \frac{1}{q^3 q^2 + \omega^2}$$

Put $Q(0) = 0$ we get

$$\{q \bar{Q}(q)\} + \frac{1}{RC} \bar{Q}(q) = \frac{V_0}{R} \frac{1}{q^3 q^2 + \omega^2}$$

$$\bar{Q}(q) = \frac{V_0}{R} \frac{1}{q^3 (q^2 + \omega^2) \left(q + \frac{1}{RC} \right)}$$

$$\bar{Q}(q) =$$

$$\frac{V_0}{R} \left\{ \left[\frac{RC}{(\omega RC)^2 + 1} \left(\frac{1}{q^3 q^2 + \omega^2} - \omega RC \frac{1}{q^2 q^2 + \omega^2} \right) \right] + \left[\frac{\omega (CR)^2}{(\omega RC)^2 + 1} \frac{1}{q^3 q + \frac{1}{RC}} \right] \right\}$$

Taking inverse Gupta Transform [5], we have

$$Q(t) =$$

$$\frac{V_0}{R} \left\{ \left[\frac{RC}{(\omega RC)^2 + 1} (\sin \omega t - \omega RC \cos \omega t) \right] + \left[\frac{\omega (CR)^2}{(\omega RC)^2 + 1} e^{-\left(\frac{1}{RC}\right)t} \right] \right\}$$

Or

$$Q(t) = \frac{V_0}{\sqrt{(\omega RC)^2 + 1}}$$

$$\left\{ \left[\frac{C}{\sqrt{(\omega RC)^2 + 1}} \sin \omega t - \frac{\omega RC^2}{\sqrt{(\omega RC)^2 + 1}} \cos \omega t \right] + \left[\frac{\omega RC^2}{\sqrt{(\omega RC)^2 + 1}} e^{-\left(\frac{1}{RC}\right)t} \right] \right\}$$

..... (7)

Put $\frac{\omega RC^2}{\sqrt{(\omega RC)^2 + 1}} = \sin \phi$ and $\frac{C}{\sqrt{(\omega RC)^2 + 1}} = \cos \phi$ such

that $\tan \phi = \omega RC$, then we can rewrite equation (7) as

$$I(t) = \frac{V_0}{\sqrt{(\omega RC)^2 + 1}}$$

$$\left\{ \left[\cos \phi \sin \omega t - \sin \phi \cos \omega t \right] + \left[\sin \phi e^{-\left(\frac{1}{RC}\right)t} \right] \right\}$$

Or

$$Q(t) = \frac{V_0}{\sqrt{(\omega RC)^2 + 1}} \left\{ \sin(\omega t - \phi) + \sin \phi e^{-\left(\frac{1}{RC}\right)t} \right\} \dots (8)$$

This equation (8) provides the complete response of the series RC network with a source of sinusoidal potential.

III. CONCLUSION

In this paper, a new integral transform called Gupta Transform has successfully applied for determining the complete response (electric current) of a series RL and RC networks with a source of sinusoidal potential. The results obtained are the same as obtained with other approaches [1-4]. This approach brought up the Gupta Transform as a new and powerful tool for determining the response of network circuits.

REFERENCES

- [1] Network Analysis M. E. Van Valkenburg. 3rd Edition. Publisher: Pearson Education, 2015.
- [2] J. S. Chitode and R.M. Jalnekar, Network Analysis and Synthesis, Publisher: Technical Publications, 2007.
- [3] Murray R. Spiegel, Theory and Problems of Laplace Transforms. Publisher: Schaum's outline series, McGraw - Hill.
- [4] Rohit Gupta, Rahul Gupta, Sonica Rajput, Convolution Method for the Complete Response of a Series L-R Network Connected to an Excitation Source of Sinusoidal Potential, International Journal of Research in Electronics And Computer Engineering, Vol. 7, issue 1, January- March 2019, pp. 658-661.
- [5] Rahul Gupta, Rohit Gupta, Dinesh Verma, Propounding a New Integral Transform: Gupta Transform with Applications in Science and Engineering, "International Journal of Scientific Research in Multidisciplinary Studies", 6(3), March (2020), pp. 14-19.
- [6] Rahul Gupta, Rohit Gupta and Dinesh Verma, Application of Novel Integral Transform: Gupta Transform to Mechanical and Electrical Oscillators, "ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS)", Volume 4, Issue 1, 2020, pp. 04-07.
- [7] Rohit Gupta, Loveneesh Talwar, Rahul Gupta, Analysis of R-L-C network circuit with steady voltage source, and with steady current source via convolution method, International journal of scientific & technology research, Volume 8, Issue 11, November 2019, , pp. 803-807.
- [8] Rohit Gupta, Yuvraj Singh and Dinesh Verma, Response of a basic series inverter by the application of convolution theorem, "ASIO Journal of Engineering & Technological Perspective Research (ASIO-JETPR)", Volume 5, Issue 1, 2020, pp. 14-17.
- [9] Rohit Gupta, Anamika Singh, Rahul Gupta, Response of Network Circuits Connected to Exponential Excitation Sources, International Advanced Research Journal in Science, Engineering and Technology, Vol. 7, Issue 2, Feb. 2020, pp.14-17.
- [10] Dinesh Verma, Yuvraj Singh, Rohit Gupta, Response of Electrical Networks with Delta Potential via Mohand Transform, International Research Journal of Innovations Engineering and Technology, volume 2, issue 2, February 2020, pp. 41-43.
- [11] P. Senthil Kumar, S. Vasuki, Applications of Aboodh Transform to Mechanics, Electrical Circuit Problems, International Journal

- for Research in Engineering Application & Management, Vol-04, Issue-06, Sep. 2018, pp. 367-369.
- [12] Rohit Gupta and Loveneesh Talwar, Elzaki Transform Means To Design A Protective RC Snubber Circuit, International Journal of Scientific and Technical Advancements, Volume 6, Issue 3, 2020, pp. 45-48.
- [13] Dinesh Verma, Rohit Gupta, Applications of Elzaki Transform To Electrical Network Circuits With Delta Function, "ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS)", Volume 4, Issue 1, 2020, pp. 21-23.
- [14] Rahul Gupta and Rohit Gupta, Impulsive Responses of Damped Mechanical And Electrical Oscillators, International Journal of Scientific and Technical Advancements, Volume 6, Issue 3, 2020, pp. 41-44.
- [15] Rohit Gupta and Rahul Gupta, Analysis of RLC circuits with exponential excitation sources by a new integral transform: Rohit Transform, "ASIO Journal of Engineering & Technological Perspective Research (ASIO-JETPR)", Volume 5, Issue 1, 2020, pp. 22-24.
- [16] Rohit Gupta, Loveneesh Talwar, Dinesh Verma, Exponential Excitation Response of Electric Network Circuits via Residue Theorem Approach, "International Journal of Scientific Research in Multidisciplinary Studies", 6(3), March (2020), pp. 47-50.
- [17] Rohit Gupta, Rahul Gupta, Sonica Rajput, Response of a parallel L- C- R network connected to an excitation source providing a constant current by matrix method, International Journal for Research in Engineering Application & Management (IJREAM), Vol. 4, Issue 7, Oct 2018, pp. 212-217.
- [18] Rohit Gupta, Yuvraj Singh Chib, Rahul Gupta, Design of the resistor-capacitor snubber network for a d. c. circuit containing an inductive load, Journal of Emerging Technologies and Innovative Research (JETIR), Volume 5, Issue 11, November 2018, pp. 68-71.
- [19] Rohit Gupta, Rahul Gupta, Matrix method for deriving the response of a series L- C- R network connected to an excitation voltage source of constant potential, Pramana Research Journal, 8(10), 2018: 120-128.
- [20] Rohit Gupta, Rahul Gupta, Sonica Rajput, Response of a parallel L- C- R network connected to an excitation source providing a constant current by matrix method, International Journal for Research in Engineering Application & Management (IJREAM), 4(7), Oct. 2018, pp. 212-217.
- [21] Rahul Gupta, Rohit Gupta and Loveneesh Talwar, Response of Network Circuits Connected To Impulsive Potential Source Via New Integral Transform: Gupta Transform, "ASIO Journal of Engineering & Technological Perspective Research (ASIO-JETPR)", Volume 5, Issue 1, 2020, pp. 18-20.
- [22] Rohit Gupta. On novel integral transform: Rohit Transform and its application to boundary value problems, "ASIO Journal of Chemistry, Physics, Mathematics and Applied Sciences (ASIO-JCPMAS)", Volume 4, Issue 1, 2020, pp. 08-13.

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