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APPLICATION OF MODIFIED LAPLACE TRANSFORM, SCALED CUBIC L TRANSFORM IN PHYSICS

Sharad B. Ugale^{1†}, Dinkar P. Patil² and Pradip R. Bhadane³

¹Research scholar, Department of Mathematics, M.V.P. Samaj's, K.R.T. Arts B.H. Commerce and A.M. Science College, Nashik, Pin 422002, Maharashtra India **Email-id** :maths.sbu@gmail.com

²Principal, Adivasi Seva Samittee's, Arts and Commerce College, Wadala, Nashik, , Pin. 422006, Maharashtra, India. **Email-id** :sdinkarpatil95@gmail.com

³Assistant Professor, M.V.P. Samaj's, Shrimati Vimlaben Khimji Tejookaya Arts, Science & Commerce College Deolali Camp, Nashik-422401. India. **Email-id** :prbhadane66@gmail.com

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Corresponding Author:

†**Sharad B. Ugale,**

Research Scholar, Department of Mathematics, M.V.P. Samaj's, K.R.T. Arts B. H. Commerce and A. M. Science College, Nashik, Pin: 422002, Maharashtra, India.

Mobile: 8055176699

Email-id :maths.sbu@gmail.com

ABSTRACT

In this study, we propose a new modification of the Laplace transform called the "SCALED CUBIC L TRANSFORM." It incorporates a cubic kernel and a scaling factor. It may be useful for solving third-order or higher differential equations. It may also handle situations where the function is stretched or compressed in time. We establish its fundamental properties, including linearity, existence, and the derivative theorem. To demonstrate its effectiveness, we apply the transform to selected problems in physics.

Keywords: Scaled Cubic L Transform, Boundary value problem, Integral transform, Laplace transformation, Ordinary differential equations, modified Laplace transform.

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1. INTRODUCTION

Differential equations are useful in solving day-to-day life problems. Therefore, solving differential equations is quite significant. A variety of methods have been proposed for solving ordinary differential equations. Among these, solving ordinary differential equations and system of ordinary differential equations with integral transforms is simple and accurate. Many integral transforms have been introduced for this purpose, including the Laplace transform [1], Fourier transform [1], a new integral transform [2], Formable transform

[3][4], the more general Sharad transform [5], the Rohit transform [6][7], new general integral transform[8], Natural transform[9], Sumudu transform, Kushare transform, Elzaki transform, HK-transform, Kamal transform etc.

In this study, we introduce a new integral transform called the Scaled Cubic L Transform. We further demonstrated its existence and other attributes using theorems.

We have discovered some Scaled Cubic L Transforms of special functions and their derivatives, which will be useful in solving problems in physics.

2. DEFINITIONS OF SCALED CUBIC L TRANSFORM (SCLT) AND INVERSE OF SCALED CUBIC L TRANSFORMS:

DEFINITION 1:

Let $g(t)$ be the exponential ordered and piecewise continuous function on the interval $[0, \infty)$ on the set \mathbb{R} .

Where,

$$E = \{g(t): \exists N > 0, M > 0, |g(t)| < Ne^{Mt}, t \in [0, \infty)\}$$

The SCLT of $g(t)$ is denoted by $T\{g(t)\}$ and is defined as follows :

$$T_u\{g(t)\} = U(s, u) = s^3 \int_0^{\infty} e^{-s^3 t} g(ut) dt$$

where $s = a + ib$ complex variable and $u > 0$.

This equation is equivalent to :

$$T_u\{g(t)\} = U(s, u) = \frac{s^3}{u} \int_0^{\infty} e^{-\frac{s^3}{u} t} g(t) dt$$

$$T_u\{g(t)\} = U(s, u) = \frac{s^3}{u} \lim_{x \rightarrow \infty} \int_0^x e^{-\frac{s^3}{u} t} g(t) dt, \quad u > 0, s > 0, s \in \mathbb{C}$$

where s and u are the transform variables, x is the real number, and the integral is taken along the line $t = x$.

DEFINITION 2:

The inverse SCLT of a function $U(s, u)$ is the original function $g(t)$. This can be written as follows.

$$T_u^{-1}[T\{g(t)\}] = g(t)$$

The inverse Scaled Cubic L Transform is denoted by $T^{-1}[U(s, u)]$ and is defined as:

$$T_u^{-1}[U(s, u)] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{s^3}{s} e^{\frac{s^3}{u} t} U(s, u) ds$$

THEOREM 1 :Sufficient conditions for the existence of the SCLT :

Let $g(t)$ be a piecewise continuous function on an interval $t \in [0, \alpha]$ and of exponential ordered M for $t > M$. Then the SCLT is exists.

Proof: Let α be any number. Then we can write

$$T_u\{g(t)\} = U(s, u) = \frac{s^3}{u} \int_0^{\alpha} e^{-\frac{s^3}{u} t} g(t) dt, = \frac{s^3}{u} \int_0^{\alpha} e^{-\frac{s^3}{u} t} g(t) dt + \frac{s^3}{u} \int_{\alpha}^{\infty} e^{-\frac{s^3}{u} t} g(t) dt,$$

But, we have given that the function $g(t)$ is piecewise continuous, then $\frac{s^3}{u} \int_0^{\alpha} e^{-\frac{s^3}{u} t} g(t) dt$ is exists.

Also, we are given that $g(t)$ is an exponential order of M . Therefore, we can write

$$\begin{aligned} \left| \frac{s^3}{u} \int_{\alpha}^{\infty} e^{-\frac{s^3}{u} t} g(t) dt \right| &\leq \left| \frac{s^3}{u} \right| \int_{\alpha}^{\infty} e^{-\frac{s^3}{u} t} |g(t)| dt \\ &\leq \left| \frac{s^3}{u} \right| \int_{\alpha}^{\infty} e^{-\frac{s^3}{u} t} N e^{Mt} dt \\ &\leq \frac{s^3}{u} N \int_0^{\infty} e^{-\left(\frac{s^3}{u} - uM\right)t} dt = \left[\frac{Ns^3}{s^3 - uM} \right] \end{aligned}$$

Thus, the SCLT is exists, if $\frac{s^3}{u} > M$.

3. METHODOLOGY: (2)

By transforming problems into a simpler form, the integral transform method finds solutions. First, a transform such as SCLT is used to transform the initial complex problem into an algebraic equation. It is easier to answer this algebraic equation. Lastly, the inverse transform is used to return the solution to the problem's original form.

4. THEOREMS AND RESULTS:

In this section, we shall prove several theorems and establish important properties of the SCLT.

THEOREM 2 (LINEARITY) :

Let $f(t)$ and $g(t)$ be the two functions whose SCLT exists and α and β be constants. Then,

$$T_u\{\alpha f(t) + \beta g(t)\} = \alpha T_u\{f(t)\} + \beta T_u\{g(t)\}$$

Proof : Using the definition of the SCLT, (3)

$$\begin{aligned} T_u\{\alpha f(t) + \beta g(t)\} &= s^3 \int_0^{\infty} e^{-s^3 t} [\alpha f(t) + \beta g(t)] dt \\ &= \alpha s^3 \int_0^{\infty} e^{-s^3 t} [f(t)] + \beta s^3 \int_0^{\infty} e^{-s^3 t} [g(t)] dt \\ &= \alpha T_u\{f(t)\} + \beta T_u\{g(t)\} \end{aligned}$$

Hence proved.

THEOREM 3: SCLT of the derivatives :

Let $g(t)$ be a differential function of order n such that its SCLT exists. Then the SCLT of its derivatives is given by

i. $T_u \left\{ \frac{dg}{dt} \right\} = \frac{s^3}{u} U(s, u) - \frac{s^3}{u} g(0)$

ii. $T_u \left\{ \frac{d^2g}{dt^2} \right\} = \left[\frac{s^3}{u} \right]^2 U(s, u) - \left[\frac{s^3}{u} \right]^2 g(0) - \frac{s^3}{u} g'(0)$

iii.
 $T_u \left\{ \frac{d^3g}{dt^3} \right\} = \left[\frac{s^3}{u} \right]^3 U(s, u) - \left[\frac{s^3}{u} \right]^3 g(0) - \left[\frac{s^3}{u} \right]^3 g'(0) - \frac{s^3}{u} g''(0)$

iv. For a positive integer n ,

$$T_u \left\{ \frac{d^n g}{dt^n} \right\} = \left(\frac{s^3}{u} \right)^n U(s, u) - \frac{s^3}{u} \sum_{k=0}^{n-1} \left(\frac{s^3}{u} \right)^{n-k} g^{(k)}(0)$$

Proof :

i. To prove this result, we use the definition of SCLT together with the integration by parts rule.

$$\begin{aligned} T_u \left\{ \frac{dg}{dt} \right\} &= \frac{s^3}{u} \int_0^\infty e^{-\frac{s^3}{u}t} \left[\frac{dg(t)}{dt} \right] dt = \frac{s^3}{u} \left[\lim_{x \rightarrow \infty} \left(e^{-\frac{s^3}{u}t} g(t) \right)_0^x + \right. \\ &\left. \frac{s^3}{u} \int_0^\infty \left(e^{-\frac{s^3}{u}t} g(t) \right) dt \right] \\ &= \frac{s^3}{u} U(s, u) - \frac{s^3}{u} g(0) \end{aligned}$$

ii. To prove this result, we use the definition of the SCLT together with the integration by parts rule

$$\begin{aligned} T_u \left\{ \frac{d^2g}{dt^2} \right\} &= \frac{s^3}{u} \int_0^\infty e^{-\frac{s^3}{u}t} \left[\frac{d^2g(t)}{dt^2} \right] dt = \\ &\frac{s^3}{u} \left[\lim_{x \rightarrow \infty} \left(e^{-\frac{s^3}{u}t} g'(t) \right)_0^x + \frac{s^3}{u} \int_0^\infty \left[e^{-\frac{s^3}{u}t} g'(t) \right] dt \right] \\ &= \left[\frac{s^3}{u} \right]^2 U(s, u) - \left[\frac{s^3}{u} \right]^2 g(0) - \frac{s^3}{u} g'(0) \end{aligned}$$

iii. To prove this result, we use the definition of the SCLT together with the integration by parts rule.

$$\begin{aligned} T_u \left\{ \frac{d^3g}{dt^3} \right\} &= \frac{s^3}{u} \int_0^\infty e^{-\frac{s^3}{u}t} \left[\frac{d^3g(t)}{dt^3} \right] dt = -\frac{s^3}{u} g''(0) + \\ &\frac{s^3}{u} \left[\frac{s^3}{u} \int_0^\infty e^{-\frac{s^3}{u}t} g''(t) dt \right] = \left[\frac{s^3}{u} \right]^3 U(s, u) - \left[\frac{s^3}{u} \right]^3 g(0) - \\ &\left[\frac{s^3}{u} \right]^2 g'(0) - \frac{s^3}{u} g''(0) \frac{s^3}{u} \left[\lim_{x \rightarrow \infty} \left(e^{-\frac{s^3}{u}t} g''(t) \right)_0^x + \right. \\ &\left. \frac{s^3}{u} \int_0^\infty \left[e^{-\frac{s^3}{u}t} g''(t) \right] dt \right] \end{aligned}$$

iv. Using mathematical induction on n , we can prove the result.

5. SCLT OF SOME SPECIFIC FUNCTIONS:

In this section, we shall determine several integral transforms of the specific functions that are particularly useful when solving differential equation by means of the SCLT.

In the following table, we present several integral transform of the specific functions, obtained using definition of SCLT and the integration by parts rule.

Table 1: SCLT of some specific functions

Sr. No.	Specific Functions $g(t)$	SCLT of Functions $T_u [g(t)]$
1	k, constant	k
2	t	$1! \left(\frac{u}{s^3} \right)$
3	t^2	$2! \left(\frac{u}{s^3} \right)^2$
4	t^3	$3! \left(\frac{u}{s^3} \right)^3$
5	t^n	$n! \left(\frac{u}{s^3} \right)^n$
6	e^{at}	$\frac{s^3}{(s^3 - ua)}$
7	e^{-at}	$\frac{s^3}{(s^3 + ua)}$
8	e^{iat}	$\frac{s^3}{(s^3 - iua)}$
9	$\sin(at)$	$\frac{uas^3}{s^6 + u^2a^2}$
10	$\cos(at)$	$\frac{s^6}{s^6 + u^2a^2}$
11	$\sinh(at)$	$\frac{uas^3}{s^6 - u^2a^2}$
12	$\cosh(at)$	$\frac{s^6}{s^6 - u^2a^2}$
13	$\delta(t) = \begin{cases} 1 & t \neq 0 \\ \infty & t = 0 \end{cases}$	$\frac{s^3}{u}$

6. APPLICATIONS OF SCLT:

In this section, we shall determine the efficiency and accuracy of the SCLT by applying it to selected problems in physics.

EXAMPLE 1 :

The vibration of mechanical system of two masses are given by the equations

$$y_1'' = -ky_1 + k(y_2 - y_1)$$

$$y_2'' = -k(y_2 - y_1) - ky_2$$

subject to the boundary conditions

$$y_1(0) = y_2(0) = 1$$

$$y_1'(0) = \sqrt{3k}, y_2'(0) = -\sqrt{3k}$$

Solution:

Applying the SCLT to equation (5) and (6), we obtain

$$\left[\frac{s^3}{u}\right]^2 T_u[y_1(t)] - \left[\frac{s^3}{u}\right]^2 y_1(0) - \frac{s^3}{u} y_1'(0) = -kT_u[y_1(t)] + kT_u[y_2(t)] - kT_u[y_1(t)]$$

$$\left[\frac{s^3}{u}\right]^2 T_u[y_2(t)] - \left[\frac{s^3}{u}\right]^2 y_2(0) - \frac{s^3}{u} y_2'(0) = -kT_u[y_2(t)] + kT_u[y_1(t)] - kT_u[y_2(t)]$$

Using the boundary conditions from equations (6) and (7), and simplifying, we obtain

$$\left[\left(\frac{s^3}{u}\right)^2 + 2k\right] T_u[y_1(t)] - kT_u[y_2(t)] = \left(\frac{s^3}{u}\right)^2 + \frac{s^3}{u} [\sqrt{3k}]$$

$$-kT_u[y_1(t)] + \left[\left(\frac{s^3}{u}\right)^2 + 2k\right] T_u[y_2(t)] = \left(\frac{s^3}{u}\right)^2 - \frac{s^3}{u} [\sqrt{3k}]$$

By adding equation (8) and equation (9), and simplifying, we obtain

$$\left[\left(\frac{s^3}{u}\right)^2 + k\right] T_u[y_1(t)] + \left[\left(\frac{s^3}{u}\right)^2 + k\right] T_u[y_2(t)] = 2\left(\frac{s^3}{u}\right)^2$$

$$T_u[y_1(t)] + T_u[y_2(t)] = \frac{2\left(\frac{s^3}{u}\right)^2}{\left[\left(\frac{s^3}{u}\right)^2 + k\right]}$$

Subtracting equation (9) from equation (8), and simplifying, we obtain

$$\left[\left(\frac{s^3}{u}\right)^2 + 3k\right] T_u[y_1(t)] - \left[\left(\frac{s^3}{u}\right)^2 + 3k\right] T_u[y_2(t)] = \frac{s^3}{u} [2\sqrt{3k}]$$

$$T_u[y_1(t)] - T_u[y_2(t)] = \frac{\frac{s^3}{u} [2\sqrt{3k}]}{\left[\left(\frac{s^3}{u}\right)^2 + 3k\right]}$$

By adding equation (10) and equation (11) and simplifying, we obtain

$$T_u[y_1(t)] = \frac{\left(\frac{s^3}{u}\right)^2}{\left[\left(\frac{s^3}{u}\right)^2 + k\right]} - \frac{\frac{s^3}{u} [\sqrt{3k}]}{\left[\left(\frac{s^3}{u}\right)^2 + 3k\right]}$$

Subtracting equation (11) and equation (10), and simplifying, we obtain

$$T_u[y_2(t)] = \frac{\left(\frac{s^3}{u}\right)^2}{\left[\left(\frac{s^3}{u}\right)^2 + k\right]} + \frac{\frac{s^3}{u} [\sqrt{3k}]}{\left[\left(\frac{s^3}{u}\right)^2 + 3k\right]}$$

After simplifying equation (11) and equation (12), we obtain the following equations (4)

$$T_u[y_1(t)] = \frac{s^6}{s^6 + ku^2} + \frac{u\sqrt{3k}s^3}{s^6 + 3ku^2} \tag{6}$$

$$T_u[y_2(t)] = \frac{s^6}{s^6 + ku^2} - \frac{u\sqrt{3k}s^3}{s^6 + 3ku^2} \tag{7}$$

Applying the inverse SCLT, we obtain the solution.

$$y_1(t) = \cos(\sqrt{k} t) + \sin(\sqrt{3k} t)$$

$$y_2(t) = \cos(\sqrt{k} t) - \sin(\sqrt{3k} t)$$

By combining above two equations, we obtain the solution of the given system of equations.

EXAMPLE 2 :

Newton's second law of impulsive force acting on a particle of a mass *m* moving along a straight line is [15]

$$m \frac{dx^2}{dt^2} = F(t) \tag{8}$$

where $F(t) = p\delta(t)$, with constant *p*, is an impulsive force acting for a short time

Obtain *x(t)* with

$$x(0) = 0 \text{ and } x'(0) = 0$$

Solution:

By applying the SCLT to the equation (14), we obtain: (10)

$$T_u\left[m \frac{dx^2}{dt^2}\right] = T_u[F(t)]$$

$$mT_u\left[\frac{dx^2}{dt^2}\right] = T_u[p\delta(t)]$$

$$\left[\frac{s^3}{u}\right]^2 T_u[x(t)] - \left[\frac{s^3}{u}\right]^2 x(0) - \frac{s^3}{u} x'(0) = \frac{p}{m} T_u[\delta(t)]$$

$$\left[\frac{s^3}{u}\right]^2 T_u[x(t)] - \left[\frac{s^3}{u}\right]^2 x(0) - \frac{s^3}{u} x'(0) = \frac{p}{m u} \dots \text{as } T_u[\delta(t)] = \frac{s^3}{u} \tag{11}$$

Using boundary conditions

$$\left[\frac{s^3}{u}\right]^2 T_{-u}[x(t)] - \left[\frac{s^3}{u}\right]^2 (0) - \frac{s^3}{u} (0) = \frac{p s^3}{m u}$$

$$\left[\frac{s^3}{u}\right]^2 T_{-u}[x(t)] = \frac{p s^3}{m u}$$

$$T_u[x(t)] = \frac{p}{m} \left[\frac{u}{s^3}\right]$$

By applying the inverse SCLT, we obtain:

$$x(t) = \frac{p}{m} t$$

$$\frac{d}{dt}[x(t)] = \frac{p}{m}$$

Thus, the effect of impulse $p\delta(t)$ is to transfer, instantaneously, p units of linear momentum to the particle.

7. DISCUSSION:

A modified version of the fundamental Laplace transform is the Scaled Cubic L Transform (SCLT). In order to solve problems when time changes or scales differently, it incorporates a scaling factor into the function and substitutes with. This facilitates the analysis of systems that behave quickly or slowly without the need for additional formulas. Certain complicated differential equations, particularly those with higher-order terms, can also be made simpler by it. In general, the SCLT is helpful for more straightforward and straightforward research of multi-scale systems.

8. CONCLUSIONS:

We present the Scaled Cubic L Transform (SCLT) and demonstrate a number of related theorems in this study. Here, we solve a few physics problems and find that this transform yields the same answers as the conventional Laplace transform, demonstrating its practical utility. Since it is still under study, it may also be useful for solving a wide range of ordinary and partial differential equations, difference equations, and integro-partial differential equations and fractional differential equations..

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